1995-96 School Year

Mathematics 33 Information Bulletin

Diploma Examinations Program

Students First! Student Evaluation

Alberta EDUCATION

This document was written primarily for:

Students	✓
Teachers	✓ Mathematics 33 teachers
Administrators	✓
Parents	
General Audience	
Others	

Distribution: Superintendents of Schools • School Principals and Teachers • The Alberta Teachers' Association • Alberta Education • General Public upon Request

Contents

NOTE

Purpose of the Bulletin

The purpose of this bulletin is to provide students and teachers of Mathematics 33 with information about the diploma examinations scheduled for 1996.

We encourage teachers to share the contents of this bulletin with students and to review the scoring criteria with them.

If you have requests, questions, or comments about the contents of this bulletin, please contact:

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Important Dates in 1996

Administration of the Mathematics 33 Diploma Examination

1996 Administrations	*Time**
Monday, January 29	9:00–11:30 A.M.
Wednesday, June 26	9:00–11:30 A.M.
Wednesday, August 14	9:00–11:30 A.M.

* The diploma examination is designed for a writing time of 2.5 h. Students may have an additional 0.5 h to complete the examination.

Scoring of the Examination

1996 Administrations	*Scoring Dates***
January	February 1 (Head Markers) February 2 to 4
June	July 9 (Head Markers) July 10 to 12
August	August 19

** Dates are tentative and will be confirmed by telephone with markers.

Notes of Interest

Information About Markers

The written-response questions of the Mathematics 33 Diploma Examinations are marked by Mathematics 33 teachers selected from those who have been recommended as markers to the Student Evaluation Branch by their superintendents. To qualify for recommendation, a teacher must have taught the complete Mathematics 33 course twice, be teaching the course in the current school year, and have an Alberta or Northwest Territories Permanent Professional Teaching Certificate.

Often, more teachers are recommended as markers by superintendents than are required by the Student Evaluation Branch for any one marking session. The following criteria are considered when markers are selected for a particular marking session:

- experience as a marker (experienced markers and first-time markers are included)
- regional representation (by zone, jurisdiction, and school)
- student population (proportional representation)

We particularly need teachers who can mark examinations written in French.

Teachers who wish to be recommended as markers for the January 1996 examination should contact their superintendents by September 22, 1995. Teachers who wish to be recommended as markers for the June and August 1996 examinations should contact their superintendents by **February 23, 1996.**

Item Writing and Field Testing

As the need arises for teachers to participate in field testing and item writing, letters are sent to superintendents requesting their nominations. **Teachers who are interested in these activities should let their superintendents know by September 22, 1995.**

Reports

After the January and June administration of the Mathematics 33 diploma examination, copies of the *Examiners' Report* for that particular administration are sent to teachers in all senior high schools in Alberta. The report contains provincial results along with comments about student performance on the examination. We welcome comments and suggestions from teachers on how to

increase the usefulness of these reports. Please direct your comments to the Mathematics 33 examination manager.

Each year, copies of a document entitled *Annual Report, Diploma Examinations Program*, are sent to each superintendent and senior high school principal in Alberta. The report contains information about the results achieved by students who wrote diploma examinations in the school year. It also contains special studies on topics of interest. The first annual report (1989–90 school year) contains a study comparing achievement in various diploma examination courses; the second (1990–91 school year) contains a study comparing the achievement of students who repeat diploma examination courses and rewrite diploma examinations with their achievement the first time and with the achievement of students who write only once. The third report (1991–92 school year) contains a study on participation rates in diploma examination courses. The fourth report (1992–93 school year) contains a study of English 30 students' application of conventions of language. The fifth report (1993–94 school year) presents a comparison of provincial participation rates in diploma examination subjects for the school years 1991–92, 1992–93, and 1993–94. Each of these special studies will be of interest to teachers who are involved in interpreting the diploma examination results of their students.

Superintendents and senior high school principals receive detailed statistical reports on how well the students in their school district did on the Mathematics 33 examination. Teachers may use these data to reflect on their students performance in the various areas of the program.

Inservices and Presentations

On a limited basis and subject to budget constraints, Student Evaluation Branch staff is available to provide inservices or presentations related to diploma examinations or the interpretation of diploma examination results.

Student Use of Scientific Calculators

Examinations are constructed to ensure that the use of particular scientific calculators neither advantages nor disadvantages individual students. Students are expected to provide their own scientific calculators.

Refer to Appendix A for the policy statement on the use of scientific calculators on diploma examinations. Students should be made aware of this policy as early as possible in the school term to ensure

they are able to use the scientific calculator of their choice when writing examinations.

Students should also be made aware of the Examination Rules, Grade 12 Diploma Examinations (see Appendix B). Students should be aware that notes stored in electronic devices must not be brought into the examination room.

Data Resources for Students

A separate data booklet will be provided for the Mathematics 33 Examination. The booklet will contain a formula sheet, 90% Box Plots, loan tables, mortgage tables, annuity tables, and present value of an annuity tables. (See Appendix E.) The formulas contained in the data booklet have been recommended by Mathematics 33 teachers across the province. Ongoing feedback in relation to the formula sheet is encouraged. Procedures for determining annuities and present value of an annuity are illustrated in Mathematics 33 curriculum standards contained in Appendix C. The Mathematics 33 Courses of Studies recommends using tables as opposed to calculating annuities using a geometric series. The banking industry of Alberta was involved in approving tables for accuracy and applicability. Laws such as allowing interest to be compounded no more frequently than semi-annually on mortgages and procedures such as including interest in the first period of an annuity have been incorporated in developing the tables. Therefore, the annuity tables provided are annuity due tables. Teachers are encouraged to review and copy the tables provided for distribution to their class in solving real-world financial problems related to the annuity, mortgage, and loan unit.

Student Requests for Rescores

Students may request a rescoring of their diploma examinations. Before applying for a rescoring, students should check their Diploma Examination Results Statement to find out the distribution of marks. The mark on the machine-scored questions is not likely to change as a result of a rescore, but the written-response mark could change slightly. Students should remember that the rescored mark will be the final mark whether it *increases* or *decreases*.

Students who decide to have an examinaton rescored must ensure that their application is received before the deadline specified on the results statement. The fee for rescoring each examination is $26.75, which includes GST. If a diploma examination mark is increased by 5% or more as a result of rescoring, the fee is refunded.

Student Requests for Rewriting

Students may rewrite a diploma examination in any regularly scheduled administration. Students must apply to the Student Evaluation Branch by November 15, 1995, and April 15, 1996, to be eligible to write the January and June diploma examinations. The tentative fee for rewriting each examination is $26.75, which includes GST. (For more details, see the *General Informaton Bulletin*.)

Mathematics 33 Information

Standards

Standards are statements that communicate how well a student needs to perform in order to be judged as having met the required learnings specified for Mathematics 33. Accordingly, the learnings for Mathematics 33 are referred to in the Mathematics 13-23-33 Program of Studies as specific knowledge, skills, and attitudes that students should have. These learnings are amplified as standards in Appendix C, Mathematics 33 Curriculum Standards. The standards are applicable to students who achieve an acceptable standard of achievement (receive a final course mark of between and including 50% and 79%) and students who achieve an excellent standard of achievement (receive a final course mark of 80% or better). Specific statements of standards in Appendix C are written primarily to inform teachers about the extent to which students must know Mathematics 33 content and must demonstrate the skills to pass the examination. Examples of questions are provided to give a profile of the acceptable and excellent standards of achievement.

Examination Specifications

The Mathematics 33 diploma examinations for 1996 will be based on the *Senior High Program of Studies for Mathematics 33*, revised May 1991.

Scoring System

The 1996 examinations will use a newly developed examination format. The examination will have both machine-scored and teacher-scored (written response) questions distributed throughout the examination.

The machine-scored questions will be of two types: multiple choice and numerical response. Examples of these can be found in Appendix C.

Each examination will have four written-response questions. Student answers to the written-response questions will be scored by teachers using both analytic and holistic scoring guides. A sample of a generalized holistic scoring guide is provided in Appendix G. The guide describes the characteristics of an answer, which corresponds to a number on a scale. The scale is appropriately proportioned to the task or question being scored and can vary between one and seven marks. Each student's answer will be scored by one teacher.

Format

The examination will have multiple-choice, numerical-response, and written-response questions. These questions will be organized into related sets. Sets are introduced by general scenarios that provide context for questions or connect large areas of understanding that students must have.

Scenarios will be identified with labels such as "Consumerism." This is a label under which questions can be applied to the real world. "Connections" is another example of a label that links questions to general mathematical abilities and understandings. Information related to solving, connecting, or introducing questions will be contained in a bordered area. **A screened bar will be used to indicate the end of a set of related questions.**

> A set of questions may contain multiple-choice and/or numerical-response and/or written-response questions.

Answer Sheet

Students will record their answers to machine-scored questions on a form similar to that found in Appendix D. Special instructions for filling in answers to numerical-response questions can be found in Appendix H.

Examination Design

The Mathematics 33 Diploma Examination is designed to reflect content as outlined in the *Senior High Program of Studies for Mathematics 33* (1991). The examination is limited to those expectations that can be measured by a paper and pencil test. Questions are grouped under scenarios and either five or six different scenarios will appear on each examination.

The following is a list of the types of scenerios that may be used on the diploma examination:

1. Alberta Industries
2. Business
3. Career Applications
4. Connections
5. Consumer Applications
6. Science and Technology
7. Sports and Recreation

The list was suggested by stakeholders involved in the development of the examination. They felt that these were areas relevant to the current program of studies and to students' post-secondary needs and opportunities.

The scenerios will present a specific context. The contexts will be related to practical problems, other school subjects or within mathematics itself. For example, forestry, agriculture, or oil and gas might be used as Alberta Industries for a practical scenerio, or mathematical relationships might be used as a Connections scenerio.

The set questions that relate to a scenerio may assess understanding from a number of core content areas.

Core Content Emphasis

The core content of the course is used to build the examination and takes into consideration, as much as possible, the recommended emphasis. The emphasis below was determined by first considering recommendations in the current Teacher Resource Manual and then considering the advice obtained from teacher item-writing committees, the responses to the information newsletter mailed province-wide in the fall of 1994, and the advice of the Mathematics 33 Diploma Examination Steering Committee (July 1995).

Core Content	*Percent Emphasis*
Powers and Radicals	10
Annuities, Mortgages, and Loans	10
Statistics	13
Trigonometry	18
Polynomial & Rational Expressions	13
Relations and Functions	18
Quadratic Functions & Equations	18

Question Format

Questions will require students to describe their method of problem solving and to communicate their descriptions of mathematical concepts and procedures.

Students will also be challenged to make mathematical connections. The "Connections" scenario will provide opportunities for students to show that they can link mathematical learnings and abilities to topics in the real world, in other disciplines, or within mathematics itself.

The design of the 1996 Mathematics 33 diploma examinations is as follows:

Question Format	*Number of Questions*	*Marks*	*Percent Emphasis*
Written Response	4	21	30
Numerical Response	12	12	17
Multiple Choice	37	37	53

Mathematical Understandings

The three mathematical understandings of concepts, procedures, and problem solving will be addressed throughout the examination. The proposed emphasis of these understandings is listed below.

	Percent Emphasis
Written Response	
Procedures, Concepts, and Problem Solving	30 (combined)
Multiple Choice and Numerical Response	
Procedures	30
Concepts	25
Problem Solving	15

Special Instructions

For further information regarding the mathematical understandings above, see Appendix F. For the descriptive words to be used on examinations, see Appendix I.

Directions for the 1996 Examinations

Expectations

In keeping with the expectations listed in the *Senior High Program of Studies for Mathematics 33*, the 1996 examinations will reflect the need to understand ideas that make up concepts and the relationships between concepts. Most importantly, the examination will reflect the need to apply understanding of concepts and procedures to solving problems. Based on a premise of what students need to learn, teacher item-writing committees, post-secondary institutions, business partners, and steering committees have suggested that the scenario format of the examination be used to reflect students' confidence and ability to transfer known mathematical skills to new or existing practical situations. This is also connected to realizing the General Learner Expectations as described in the *Senior High Program of Studies for Mathematics 33*.

Scenerios

Students will be required to realize the general learner expectations. In realizing these expectations, scenarios will be used wherein students will need to use mathematical understandings to analyze, investigate, interpret, and make decisions related to real-world phenomena (future endeavors, employment, and everyday life). As well, students will be required to make connections among the various mathematical concepts and between mathematics and other disciplines.

Abstract and Concrete Understandings

For the examination students will be required to demonstrate both abstract and concrete understandings. An example of a concrete understanding is being able to apply knowledge gained in the Annuity, Mortgage, and Loan Unit to one's personal finance or to operating a small business. Abstract understandings are seen, for

example, as being used to connect knowledge of quadratic functions to their graphical representations. Students will be expected to demonstrate both concrete and abstract understandings of the subject matter in any unit. Concrete understanding in quadratics would relate to applying an equation or graph to a real-world situation. As a consequence, scenarios will have either an applied or a pure mathematics flavour.

Communication

Teacher response to the information newsletter questionnaire in the fall of 1994 and response from item writing committees endorsed the general learner expectation that students be able to communicate mathematical ideas effectively. Stakeholders have indicated that mathematical literacy is a current skill for employability. For these reasons, the open-ended questions will require students to communicate answers effectively in writing. Students may be required to explain a solution, articulate their reasoning, describe a mathematical situation, write directions to a problem, pose questions, and explain new understandings formulated.

Problem Solving

Effectively communicating solutions will enhance student grades on the written-response component of the examination. It should be noted, however, that the written-response questions will focus on students' understanding of the problem-solving process. Students will be rewarded for selecting and carrying through a problem-solving strategy.

Appendices

Appendix A

Calculator Policy

POLICY: USE OF CALCULATORS ON ALBERTA EDUCATION DIPLOMA EXAMINATIONS

Background

The knowledge, skills, and attitudes relevant to technology and its uses are being incorporated into courses and programs of study wherever appropriate. Students are expected to learn the advantages and limitations of technological developments and their impact upon society. The ability to use technology helps students understand and appreciate the process of technological change, gives added depth to programs, and provides the basis for the development of skills and understanding. These expectations are reflected in the diploma examinations. Since the data provided for writing diploma examinations in mathematics and the sciences do not include information such as logarithms and trigonometric functions, students will need to use scientific calculators for these exams.

Definition

This policy considers a scientific calculator to be a hand-held device designed primarily for mathematical computations. Included in this definition are those scientific calculators having graphing capabilities, built-in formulas, mathematical functions, or other programmable features.

Policy

To ensure compatibility with provincial *Programs of Study* and equity and fairness for all students, Alberta Education expects students to use scientific calculators, as defined above, when they are writing diploma examinations in mathematics and the sciences. Examinations are constructed to ensure that the use of particular models of calculators neither advantages nor disadvantages individual students.

Procedures

1. Teachers must, at the beginning of a course, advise students of the types of calculators that they may use when writing diploma examinations in mathematics and the sciences. Teachers must also advise students of the types of information that can be stored in calculators that are brought into diploma examinations. Calculators that have built-in notes (definitions or explanations in alpha notation) that cannot be cleared are not permitted.

2. Students must clear calculators that are brought into diploma examinations of all information they have stored except for
   a. programs used for computing values of the formulas on the diploma examination data tear-out pages or in the data booklets,
   b. programs used for graphing quadratic relations as found in the Mathematics 30/33 Interim Teacher Resource Manual.

3. Students must not bring external devices to support calculators into the exam. Such devices include manuals, printed or electronic cards, printers, memory expansion chips or cards, external keyboards, or any annotations outlining operational procedures for scientific calculators.

4. In preparation for calculator failure, students may bring extra calculators and batteries into the exam room.

5. During exams, supervising teachers must ensure that
   a. all calculators operate in silent mode,
   b. students do not share calculators or information contained within them,
   c. calculator cases are stored on the floor throughout the exam, and
   d. all examination rules are followed.

6. If you have any questions or comments about the implementation of this policy, please contact the Math/Science Unit, Student Evaluation Branch, at 403-427-0010 or FAX 403-422-4200.

Appendix B

Examination Rules

GRADE 12 DIPLOMA EXAMINATIONS

1. **Admittance to the Examination Room**
   Students must not enter or leave the examination room without the consent of the supervising teacher.

2. **Student Identification**
   Students must present identification that includes their signature and photograph. One of the following documents is acceptable: driver's licence, passport, or student identification card. Students must not write an examination under a false identity or knowingly provide false information on an application form.

3. **Identification on Examinations**
   Students must not write their names or the name of their school anywhere in or on the examination booklet, other than on the back cover.

4. **Time**
   Students must write an examination during the specified time and may not hand in a paper until at least one hour of the examination time has elapsed. Students who arrive more than one hour after an examination has started will not be allowed to write the examination. Students who arrive late but within the first hour of an examination sitting may be allowed to write only at the discretion of the supervising teacher.

5. **Discussion**
   Students must not discuss the examination with the supervising teacher unless the examination booklet is incomplete or illegible. Students must not talk, whisper, or exchange information with one another.

6. **Answer Sheets**
   Students must use an HB pencil to record their answers on the machine-scorable answer sheets.

7. **Materials Allowed:**
   - ***English 30, English 33:***
     For Part A only, all students may use a print or electronic dictionary (English language or translation), a thesaurus, and an authorized writer's handbook (for example: *The Canadian Writer's Handbook*, Prentice Hall).
   - ***Français 30:***
     For Partie A only, all students may use a print or electronic dictionary (French language or translation), a thesaurus, and a book of verb forms/tenses.
   - ***Biology 30, Mathematics 30, Physics 30:***
     Tear-out data pages are provided in the examination booklets. Students need to use scientific calculators, but must not share them.
   - ***Chemistry 30, Mathematics 33, Science 30:***
     Students must use the separate data booklet provided for each of these examinations. Students need to use scientific calculators, but must not share them.
   - ***All mathematics and science examinations:***
     Students may use calculator programs designed to perform mathematical computations and/or those designed to assist students in graphing.

8. **Materials Not Allowed:**
   - ***All Subjects:***
     Students must not bring any papers, notes of any kind, books (other than those allowed, see #7) into the examination room. Students must ensure that dictionaries (print and electronic), references, and calculators are completely free of notes or study material.
   - ***Biology 30, Chemistry 30, Mathematics 30, Mathematics 33, Physics 30, Science 30, Social Studies 30, Social Studies 33:***
     Reference materials and dictionaries (translation, English and/or French language) are not allowed.

Appendix C Mathematics 33 Standards and Sample Questions

Contents

Curriculum Standards

Curriculum standards are criteria that can be used to judge when a student has met the expectations of the Program of Studies for a course.

Curriculum standards are statements of what students are expected to know and be able to do. These statements are made in the Program of Studies for Mathematics 33 and have been enlarged upon in this document in order to set the stage for the development of the Diploma Examination. The statements made here represent the Program of Studies and have been organized to include general outcomes for each unit and specific outcomes for each unit. The specific outcomes have been further divided into expectations that will be held for students who demonstrate **acceptable** and **excellent** achievement.

Acceptable Achievement

Students who demonstrate acceptable achievement but not excellent achievement in Mathematics 33 will receive a final course mark between and including 50% and 79%. Typically, these students have gained new skills and knowledge in mathematics but can anticipate difficulties if they choose to enroll in post-secondary mathematics courses. They have demonstrated mathematical skills and knowledge in seven content strands of the Mathematics 33 curriculum and exhibit an ability to apply a range of problem-solving skills to these content strands.

Excellent Achievement

Students who demonstrate excellent achievement will receive a final course mark of 80% or higher. Such students have demonstrated their ability and interest in mathematics and feel confident about their mathematical abilities. These students should encounter little difficulty in post-secondary mathematics programs; they should be encouraged to pursue careers in which they will use their talents in mathematics.

Statements

The specific statements of standards that follow were written primarily to inform Mathematics 33 teachers about the extent to which students must know the Mathematics 33 content and must demonstrate the required skills to pass the examination.

Examples

A limited number of examples are provided for each section of the course. The questions provided are by no means exhaustive and have been coded to reflect the acceptable and excellent outcomes and are indicative of the type of questions that students will be presented with in the formal examination. Many of the questions have been contextualized in scenarios that describe a mathematical situation or present information that will be used in questions.

Relations and Functions

Students who achieve the standards can demonstrate the following general outcome upon completion of the unit:
Apply the concepts within relationships and functions to describe real-world phenomena that can be expressed in two variables.

Illustration of the general outcome
Given an everyday situation, students can construct a graphical model of it, analyze it, and then generalize the model to other situations.

The student demonstrating acceptable achievement can:
- interpret, orally or in writing, a graph that models a real-world situation
- create a graph that models a real-world situation
- identify the dependent and independent variables from the graph of a function
- explain why the independent variable is independent in a given context
- describe, orally or in writing, the domain and range of any function, except the reciprocal function, by examining its graph or a set of ordered pairs
- interpolate from graphs of continuous functions
- extrapolate from graphs of linear functions
- verify whether a point is or is not on the graph of a function
- determine the values of a function given particular domain values
- recognize, illustrate, and classify linear, quadratic, cubic, absolute value, first-degree reciprocal, radical, and exponential functions
- determine the inverse of a function when provided with the graph of the function
- explain and show the effect of a single parameter, a, b, or c on the graph of $y = c \bullet f(x - a) + b$
- participate in and contribute toward the problem-solving process for problems that require the use of functions studied in Mathematics 33

The student demonstrating excellent achievement can:

- state the domain and range of any function using mathematical notation
- illustrate the relationship between two variables in an everyday situation using a defining rule
- determine whether or not a cause-and-effect relationship exists between two variables, and in the case of such a relationship, decide which of the two variables is the independent variable
- extrapolate from graphs of any functions
- given particular values for a function, determine the corresponding values of the domain
- recognize and classify real-world situations that are depicted as being a linear, quadratic, cubic, absolute value, first-degree reciprocal, radical, or exponential function
- determine the inverse of a function
- explain, orally or in writing, and show the effects of the parameters a, b, and c on the graph of $y = c \cdot f(x - a) + b$
- find the complete solution to problems that require the use of functions studied in Mathematics 33

Following are examples of questions from the Relations and Functions Unit that illustrate the standards under the scenario of consumerism.

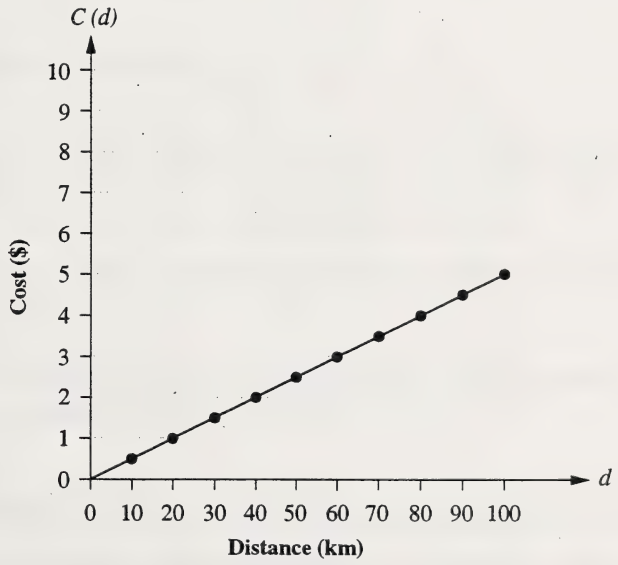
Note: The format used to number multiple-choice, numeric-response, and written-response questions on examinations and field tests is illustrated in the following section.

The student demonstrating acceptable achievement can answer the following types of questions.

SCENARIO: **CONSUMERISM**
Aaron used his knowledge of relations and functions to analyze costs associated with operating a car.

Use the following information to answer the next two questions.

Aaron created the following graph relating the cost of fuel to the distance travelled.



1. Aaron's graph may be considered as being part of the graph of a

A. cubic function
* **B.** linear function
C. quadratic function
D. reciprocal function

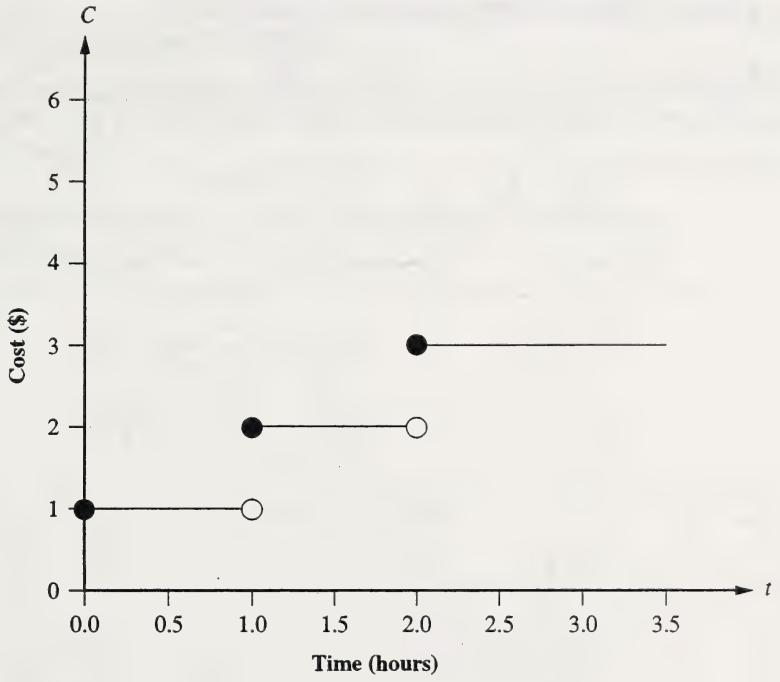
Numerical Response

1. The predicted cost of a 90 km ride, using the graph provided, would be $\$\underline{\quad 4.50 \quad}$.

(Record your answer to the nearest dollar.)

Use the following information to answer the next question.

The graph below represents the cost, C, in dollars, that Aaron was charged for various lengths of parking time, t, in hours, at a parkade.



Written Response

1. Write a story describing the amount of money Aaron would pay in the three different time intervals shown.

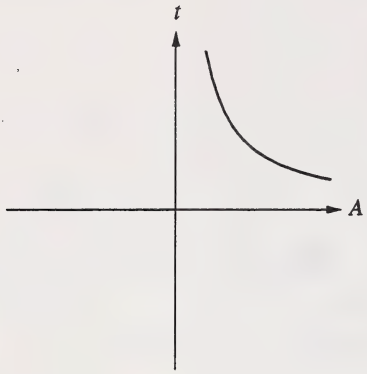
Possible Solution:

Aaron enters the parkade and automatically owes the attendant $1.00. If Aaron leaves the parkade before one hour elapses, the amount owing will still be $1.00. After one hour, Aaron's cost jumps to $2.00. Aaron owes $2.00 for any time between one and two hours. After two hours, Aaron's cost jumps to $3.00 and this appears to be the maximum cost that Aaron would pay no matter how long the car is parked.

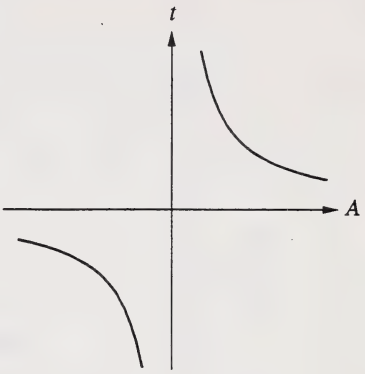
The student demonstrating excellent achievement can answer the following type of question.

2. The time t needed to fill a tank varies inversely with the cross-sectional area, A, of the hose. Aaron linked filling the gas tank of a car to one of the following graphs. Which graph **best** illustrates this relationship?

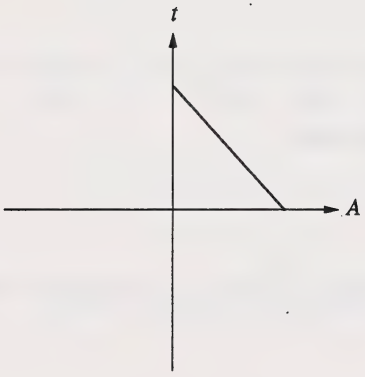
***A.**



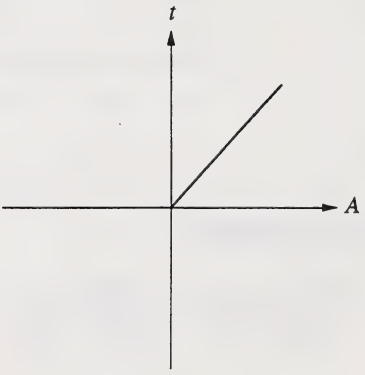
B.



C.



D.

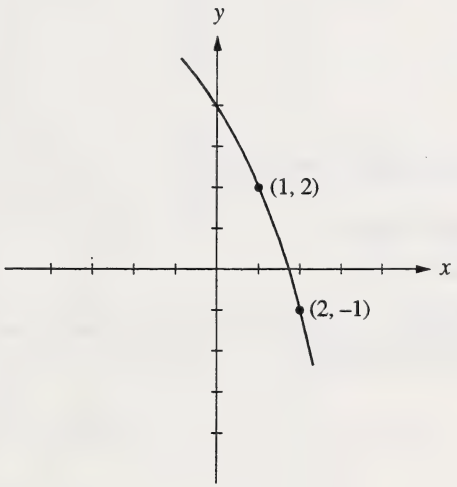


The following questions also illustrate standards from the Relations and Functions Unit but under the scenario of connections.

The student demonstrating acceptable achievement can answer the following types of questions.

SCENARIO: **CONNECTIONS**
Your understanding of the relationships among variables and equations can be connected to understanding of graphical representations in answering the next questions.

3. The graph of the function $y = f(x)$ is shown below. Two points on $y = f(x)$ are $(1, 2)$ and $(2, -1)$.



An ordered pair that represents a point on the graph of the inverse of this function is

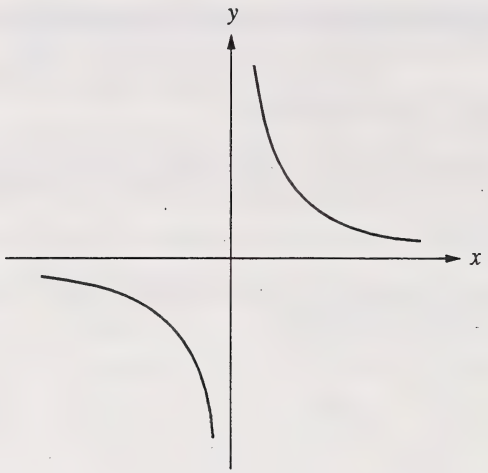
A. $\left(-1, \frac{1}{2}\right)$

* **B.** $(-1, 2)$

C. $(-2, 1)$

D. $\left(1, \frac{1}{2}\right)$

4. The graph of a function is shown below.



The function represented by the graph is

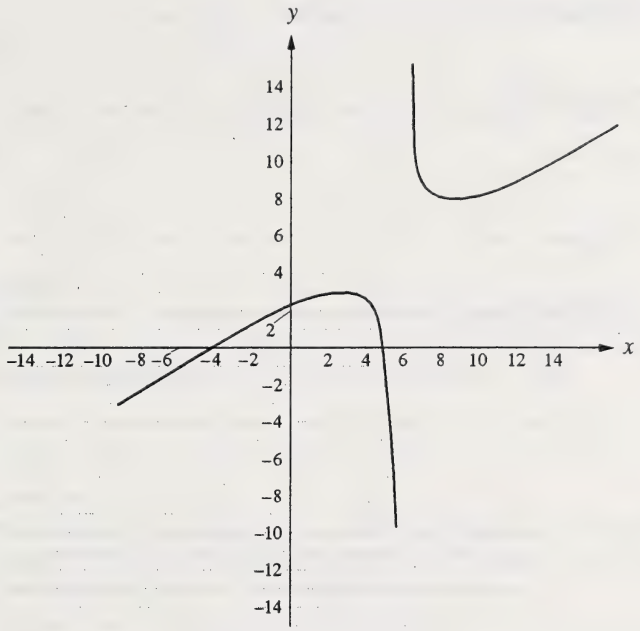
A. an exponential function

* **B.** a reciprocal function

C. a cubic function

D. an absolute value function

5. The graph of a function $y = f(x)$ is shown below.



The domain and range, respectively, of the function shown are

* **A.** $D = \{x \mid x \neq 6\}$
$R = \{y \mid y \leq 3 \text{ or } y \geq 8\}$

B. $D = \{y \mid y \neq 6\}$
$R = \{x \mid x \leq 3 \text{ or } y \geq 8\}$

C. $D = \{x \mid x \neq 6\}$
$R = \{y \mid y \in \text{Real numbers}\}$

D. $D = \{x \mid x \in \text{Real numbers}\}$
$R = \{y \mid y \in \text{Real numbers}\}$

The student demonstrating excellent achievement can answer the following types of questions.

6. The inverse function of $f(x) = 3x - 2$ is

* **A.** $f^{-1}(x) = \frac{x+2}{3}$

B. $f^{-1}(x) = \frac{1}{3}x - \frac{1}{2}$

C. $f^{-1}(x) = -3x + 2$

D. $f^{-1}(x) = x + 2$

Written Response

The function $f(x) = x^3$ is transformed so that a change in the parameters a, b, and c in $y = c \bullet f(x-a) + b$ cause it to become $f(x) = -1(x-3)^3 + 2$.

1. Describe in detail the effect of changing each parameter a, b, and c on the graph of $f(x) = x^3$.

Parameter a
Possible Solution: The parameter a shifts the graph to the right a units if a is positive and to the left a units if a is negative. In this case, the value of a causes the graph of $f(x) = x^3$ to be shifted right 3 units.

Parameter b
Possible Solution: The parameter b shifts the graph up b units if b is positive and down b units if b is negative. In this case, the value of b causes the graph of $f(x) = x^3$ to be shifted up 2 units.

Parameter c
Possible Solution: Having a negative value for c reflects the graph in the x-axis.

Quadratic Functions and Equations

Students who achieve the standards can demonstrate the following general outcome upon completion of the unit: *Connect variables and equations to graphical representations of quadratic functions that model real-world contexts.*

Illustration of the general outcome
Given any quadratic function, either directly or implied in a problem, and the equation of the function, students can determine the graph of the function and the roots of the corresponding equation, **and** can describe, in writing, the relationship between the roots and the graph.

The student demonstrating acceptable achievement can:

- sketch the graphs of quadratic functions $y = a(x - h)^2 + k$, where h and k are integers, a is a rational number, and $a \neq 0$
- transform $y = ax^2 + bx + c$ to $y = a(x - h)^2 + k$, where h and k are integers and a is a rational number, $a \neq 0$
- given the graph or the equation of a quadratic function, determine its vertex, axis of symmetry, domain, range, maximum or minimum value of the function, and x - and y-intercepts
- given a real-world situation modelled by a quadratic function, place into the appropriate context the maximum or minimum values and the x - and y-intercepts
- solve quadratic equations whose solutions are elements of the real numbers
- determine whether or not a given quadratic equation describes a particular real-world situation
- recognize that there is a relationship between the x-intercepts of the graph of a quadratic function, the roots of its corresponding equation, and the zeros of the quadratic function
- solve a radical equation with no extraneous roots consisting of a single radical whose solutions are the roots of a quadratic equation
- solve an equation, which includes two rational expressions with monomial denominators, whose solution(s) is/are the root(s) of a quadratic or a linear equation
- solve problems that involve equations with radicals or rational expressions, whose solutions are the roots of quadratic equations, when the radical or rational equation is given

- solve simple problems whose solutions are the roots of quadratic equations
- use the x-intercepts of a quadratic function, where $c = \pm 1$ in $c(x-a)(x-d)$, to determine a corresponding quadratic equation

The student demonstrating excellent achievement can:

- transform $y = ax^2 + bx + c$ to $y = a(x-h)^2 + k$, where h and k are rational and $a \neq 0$
- given any quadratic function, determine the vertex, axis of symmetry, domain, range, maximum or minimum value x - and y-intercepts, and its sketch
- derive a quadratic function that models a real-world situation and place into the appropriate context the maximum or minimum values and the x - and y-intercepts
- explain, orally or in writing, the relationship between the x-intercepts of the graph of a quadratic function, the roots of its corresponding equation, and the zeros of the quadratic function
- solve a radical equation that may have extraneous roots with two or more like radicals and whose solution is the roots of a quadratic equation
- solve a rational expression equation whose solution are the roots of a quadratic or linear equation
- solve problems that involve equations with radicals or rational expressions whose solutions are the roots of quadratic equations

The student demonstrating the acceptable achievement can answer the following types of questions.

SCENARIO: **CONNECTIONS**
Your understanding of the relationships between equations and graphs will be useful in solving the next set of questions.

Numerical Response

1. The smallest real number solution (to the nearest tenth) of the quadratic equation $2x^2 - 11x + 15 = 0$ is _____2.5_____ .

(Record your answer on the answer sheet.)

Solution:

$$2x^2 - 11x + 15 = 0$$
$$(2x - 5)(x - 3) = 0$$
$$2x - 5 = 0 \quad \text{OR} \quad x - 3 = 0$$
$$2x = 5 \qquad\qquad x = 3$$
$$x = \frac{5}{2} \text{ or } 2.5$$

Therefore, the smallest real number solution rounded to the nearest tenth is **2.5**.

Numerical Response

2. For the equation $\sqrt{15x+1} - 3 = x$, the largest numerical solution is _____8.0_____.

(Record your answer on the answer sheet.)

Solution:

$$\begin{aligned} \sqrt{15x+1} &= x+3 \\ 15x+1 &= x^2+6x+9 \\ 0 &= x^2+6x-15x+9-1 \\ 0 &= x^2-9x+8 \\ 0 &= (x-8)(x-1) \\ x-8 &= 0 \quad \text{OR} \quad x-1 = 0 \\ x &= 8 \quad \text{OR} \quad x = 1 \end{aligned}$$

Therefore, the correct answer is 8.

Connect your ability to write a quadratic equation to answering the next question.

1. The floor in a warehouse is rectangular in shape and has an area of $28\ m^2$. The length is $3\ m$ longer than the width. If x represents the measure of the width, then an equation that could be used to find the measure of the width is

A. $x^2 - 25 = 0$

B. $4x^2 - 28 = 0$

* **C.** $x^2 + 3x - 28 = 0$

D. $x^2 + 3x + 28 = 0$

Quadratic functions are sometimes represented by graphs on a coordinate plane. Connecting your understanding of quadratic functions to their graphs will be helpful in answering the next three questions.

2. The equation of a quadratic function having a vertex at $(-1, 3)$ is

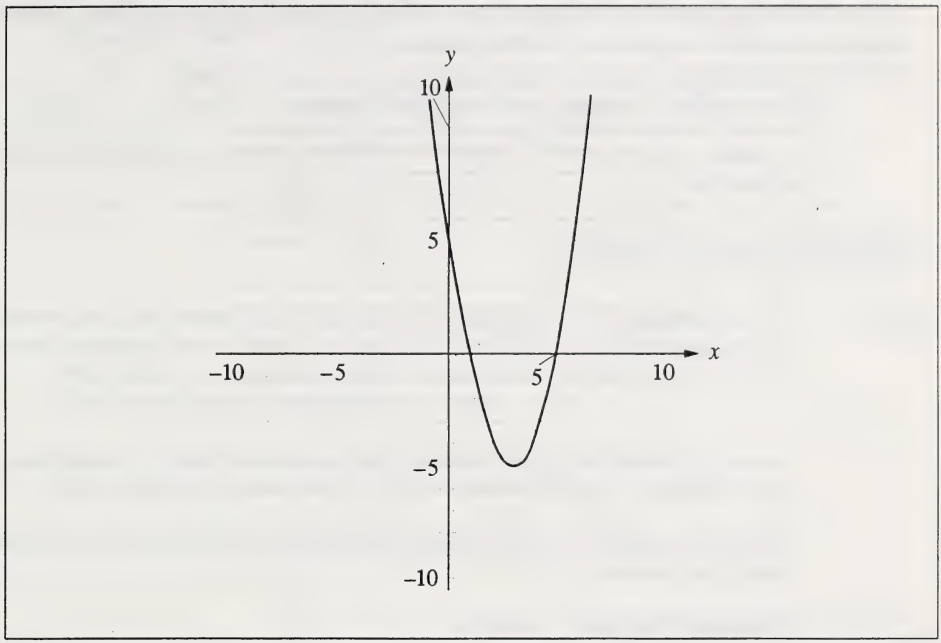
A. $y = (x - 1)^2 + 3$

B. $y = (x - 1)^2 - 3$

* **C.** $y = (x + 1)^2 + 3$

D. $y = (x + 1)^2 - 3$

Use the following graph to answer questions 3 and 4.



3. The graph above represents a quadratic function. The vertex of the graph is

A. (0, 4)
B. (5.5, 0)
C. (–5, 3)
* **D.** (3, –5)

4. The minimum/maximum value of the function represented by the graph above is a

* **A.** minimum of –5
B. maximum of –5
C. minimum of 3
D. maximum of 3

The student demonstrating acceptable achievement can successfully complete part of the following question, whereas the student demonstrating excellent achievement can successfully complete all of this question.

SCENARIO: **SPORTS AND RECREATION**
In a math project, Bobbie connected understanding of quadratics to diving and football.

Written Response – 4 marks

1. **a.** When Bobbie dove from the spring diving board at the local swimming pool, the path of his dive could be approximated by the quadratic function

$$H(t) = -5(t-1)^2 + 6$$

where H is the height of Bobbie's toes above the water in metres and t is the time elapsed in seconds after Bobbie's toes leave the diving board.

Find the maximum height Bobbie's toes reach and the time at which this height is reached.

Possible Solution: (2 marks)

Maximum height of Bobbie's toes occurs at the vertex $(1, 6)$.

From the vertex, the height is 6 m when the time (t) is 1 second.

OR

Bobbie's toes reached maximum height of 6 m when the time was 1 second.

b. Find the height of the diving board.

Possible Solution: (2 marks)

The height of a diving board corresponds to the point where no time has elapsed, so set $t = 0$.

$$\begin{aligned} H(0) &= -5(0-1)^2 + 6 \\ &= -5(1) + 6 \\ &= 1 \end{aligned}$$

The height of the diving board is approximately 1 metre.

c. Find the elapsed time after which Bobbie's toes touched the water.

Possible Solution: (2 marks)

The time that Bobbie's toes enter the water corresponds to the point at which height = 0. So, set $H = 0$ and solve for t.

$$\begin{aligned} -5(t-1)^2 + 6 &= 0 \\ -5(t^2 - 2t + 1) + 6 &= 0 \\ -5t^2 + 10t - 5 + 6 &= 0 \\ -5t^2 + 10t + 1 &= 0 \\ t &= -0.1 \text{ or } 2.1 \end{aligned}$$

Neglecting the negative time, Bobbie's toes touched the water at approximately 2.1 seconds.

The student demonstrating excellent achievement can answer the following types of questions.

5. A football is kicked upward and travels in a parabolic path. At the instant the football reaches the path's maximum height of 25 m, the ball has travelled a horizontal distance of 50 m. A **possible** function for the path relating height, h, to a horizontal distance, x, is

A. $h(x) = 5(x-25)^2 + 50$

B. $h(x) = -5(x-25)^2 + 25$

C. $h(x) = -5(x-25)^2 + 50$

* **D.** $h(x) = 5(x-50)^2 + 25$

In math class Bobbie had to explain the relationship between a quadratic function and a quadratic equation.

Written Response – 4 marks

2. Show that the solution of the quadratic equation $x^2 + 5x = 6$ is numerically identical to the x-intercept(s) of the graph of the quadratic function $y = x^2 + 5x - 6$, and explain **why** they are the same.

First part of solution: (to the quadratic equation)
The quadratic equation $x^2 + 5x = 6$ may be rewritten as

$$x^2 + 5x - 6 = 0$$
$$(x+6)(x-1) = 0$$
$$x+6 = 0 \quad \text{or} \quad x-1 = 0$$
$$\text{So, } x = -6 \quad \text{or} \quad x = 1$$

Second part of solution: (x-intercepts of the graph of the quadratic function)
In the quadratic function $y = x^2 + 5x - 6$, the x-intercepts of the function occur at points where the graph crosses or touches the x-axis and $\therefore$ $y = 0$ at these points.

So: set $y = 0$.
This results in the quadratic equation $0 = x^2 + 5x - 6$, which is the same equation as in the first part above and $\therefore$ has the same solutions.
$\therefore$ the x-intercepts of the function $y = x^2 + 5x - 6$ are -6 and 1, which correspond to the roots of the quadratic equation $x^2 + 5x - 6 = 0$.

Powers and Radicals

Students who achieve the standards can demonstrate the following general outcome upon completion of the unit: *Determine equivalent forms of radical expressions. Solve and verify radical equations.*

Illustration of the general outcome
Show that mathematical operations on radicals are similar to the mathematical operations on polynomials.

The student demonstrating acceptable achievement can:

- perform the operations of addition, subtraction, multiplication, and division on second-order radicals with monomial demoninators and numerical radicands, and show that these operations are a specific instance of the operations on polynomials
- change second-order radicals from mixed to entire form and vice-versa
- solve second-order radical equations

The student demonstrating excellent achievement can:

- change third-order radicals from mixed to entire form and vice-versa
- solve second-order radical equations whose solutions involve the simplification of a quadratic equation
- solve problems involving radical equations

The student demonstrating acceptable achievement can answer the following types of questions.

SCENARIO: **CAREER APPLICATIONS**
Part of Dali's on-the-job training with Transworld Utilities involved the review of radicals. Transworld Utilities supplies electrical power, and employees are asked to solve problems that often involve radicals. Help Dali by answering the following questions.

1. The correct simplified version of the expression $3\sqrt{5} + 7\sqrt{2} - 5\sqrt{5} + 2\sqrt{2}$ is

A. $8\sqrt{52} + 9\sqrt{2}$

B. $-17\sqrt{10}$

* **C.** $-2\sqrt{5} + 9\sqrt{2}$

D. $7\sqrt{10}$

2. An equivalent form of the expression $3\sqrt{98}$ is

A. $10\sqrt{2}$

* **B.** $21\sqrt{2}$

C. $6\sqrt{49}$

D. $5\sqrt{49}$

Before accepting the job and training at Transworld Utilities, Dali applied to a police force and was given the following problem to solve.

Numerical Response

1. At the scene of an accident, police measure the approximate speed that a vehicle was travelling by measuring the length of the skid marks left on the pavement. One formula used for this purpose is $v = 14.9\sqrt{L} - 20.4$, where v represents the speed of the vehicle, and L represents the length of the skid marks in metres. A vehicle is travelling on a road at 55 km/h and brakes so that it skids to a stop. The length of the skid mark (to one decimal place) that the vehicle will leave on the road is __25.6__ m.

(Record your answer on the answer sheet.)

Solution:

$$55 = 14.9\sqrt{L} - 20.4$$

$$75.4 = 14.9\sqrt{L}$$

$$\frac{75.4}{14.9} = \sqrt{L}$$

$$25.6 = L$$

The length of the skid mark, to one decimal place, is 25.6 metres.

The student demonstrating excellent achievement can answer the following types of questions.

Numerical Response

2. The largest root to the equation $\sqrt{x}+4=x-8$ is ____16____.

(Record your answer on the answer sheet.)

Solution:

$$\sqrt{x}+4 = x-8$$
$$\sqrt{x} = x-12$$
$$\left(\sqrt{x}\right)^2 = (x-12)^2$$
$$x = x^2-24x+144$$
$$0 = x^2-25x+144$$
$$0 = (x-9)(x-16)$$
$$x = 9 \quad \text{OR} \quad x=16$$

$x = 9$ is not a solution.

The largest root is 16.

3. The radical $2\sqrt[3]{5}$ is equivalent to

A. $\sqrt[3]{10}$

B. $\sqrt[3]{25}$

C. $\sqrt[3]{32}$

* **D.** $\sqrt[3]{40}$

Polynomials and Rational Expressions

Students who achieve the standards can demonstrate the following general outcome upon completion of the unit: *Show how the operations on rational numbers can be extended to rational expressions and the applications of rational expressions.*

Illustration of the general outcome
Show that the operations on rational expressions are a generalization of the operations on rational numbers.

The student demonstrating acceptable achievement can:

- recognize and give examples of rational expressions
- determine the real non-permissible values of a variable in a rational expression with its denominator in the form of a monomial, binomial, or trinomial of the form $x^2 + bx + c$ or with its denominator in factored form
- recognize and give examples of rational expressions where there are no real non-permissible values for the variable
- simplify rational expressions that contain monomials, binomials, or trinomials in the form $ax^2 + bx + c$ but that do not contain factors of – 1
- multiply or divide two rational expressions that contain monomials, binomials, or trinomials in the form $ax^2 + bx + c$
- add or subtract two rational expressions with non-factorable binomial denominators
- solve equations involving rational expressions with non-factorable denominators
- solve problems involving rational expressions given the equation

The student demonstrating excellent achievement can:

- determine the real non-permissible values of a variable in any rational expression
- describe the relationship between non-permissible values of a rational expression and the graph of the rational function (and how this relates to the reciprocal function studied in relations and functions)
- simplify rational expressions that do contain factors of –1
- multiply or divide two or more rational expressions
- add or subtract two rational expressions
- solve equations involving rational expressions with factorable denominators
- solve problems involving equations containing rational expressions

The student demonstrating acceptable achievement can answer the following types of questions.

SCENARIO: **CONNECTIONS**
Kelly solved the following questions by connecting factoring, equation-solving skills, and operations on fractions to rational expressions and equations.

Numerical Response

1. The largest non-permissible value of x for the rational expression $\frac{x+4}{x^2+x-6}$ is _____2_____.
(Record your answer on the answer sheet.)

2. A simplified form of $\frac{1}{m-1} + \frac{m+1}{2(m-1)}$ is

A. $\frac{m+3}{2}$

B. $\frac{m+2}{2(m-1)}$, $m \neq 1$

* **C.** $\frac{m+3}{2(m-1)}$, $m \neq 1$

D. $\frac{2m+3}{2}$

The student demonstrating excellent achievement can answer the following types of questions.

2. A simplified form of $\frac{x+y}{y-x} \div \frac{1}{x-y}$ is

A. $-x+y,\ x \neq y$

B. $x+y,\ x-y \neq 0$

* **C.** $-x-y,\ y-x \neq 0$

D. $x-y,\ y \neq x$

3. When fully simplified, $\frac{2x}{x^2-4} + \frac{x-2}{x^2-4x+4}$ is equal to

* **A.** $\frac{3x+2}{(x-2)(x+2)},\ x \neq \pm 2$

B. $\frac{2x}{(x-2)(x+2)},\ x \neq \pm 2$

C. $\frac{x^2+2x-4}{(x-2)(x+2)},\ x \neq \pm 2$

D. $\frac{2x+1}{(x-2)(x+2)},\ x \neq \pm 2$

Written Response – 4 marks

1. A small plane and a transport truck travel 300 km from Calgary to Edmonton. The truck travels 50 km/h slower than the small plane. The truck takes 1 hour longer to travel the same distance. Calculate the current speed of the small plane given Kelly's partial solution.

Kelly's Partial Solution: Let s = the speed of the small plane.

	Distance	Speed	Time
Small plane	300	s	$\frac{300}{s}$
Truck	300	$s - 50$	$\frac{300}{s - 50}$

Time for transport truck – Time for small plane = 1

Completed Solution:

$$\frac{300}{s-50} - \frac{300}{s} = 1$$

$$300s - 300s + 15\,000 = 1(s)(s - 50)$$

$$0 = s^2 - 50s - 15\,000$$

$$0 = (s + 100)(s - 150)$$

$$\therefore\ s + 100 = 0 \quad \text{OR} \quad s - 150 = 0$$

$$s = -100 \quad \text{OR} \quad s = 150$$

Neglecting the negative value, the current speed of the small plane is 150 km/h.

Trigonometry

Students who achieve the standards can demonstrate the following general outcome upon completion of the unit: *Solve problems that involve triangles and their applications.*

Illustrations of the general outcome
Extend right angle trigonometry to angles in a coordinate plane.

Use understanding of sine and cosine law to solve oblique triangles.

The student demonstrating acceptable achievement can:

- recognize and give examples of the principal and other co-terminal angles for angles drawn in standard position on a coordinate plane
- determine the measure of the reference angle for any angle in standard position on a coordinate plane
- determine the sine, cosine, and tangent ratios, and the measure of the principal angle for coordinates of any point with numerical coordinates on the terminal arm of an angle in standard position
- determine the sine, cosine, and tangent ratios for any angle measure
- recognize the graph of $y = \sin\theta$
- use the graph of $y = \sin\theta$ to solve trigonometric equations
- describe, orally or in writing, the effects of changing either of the parameters a or b on the graph of the function $y = a \sin b\theta$
- determine the measures of sides and angles in problems that involve two right triangles if a diagram is given
- decide whether or not the information provided for a triangle uniquely defines an oblique triangle, and if it does not, construct the possible triangles
- determine the missing measures of sides and angles in problems that involve oblique triangles if a diagram is given
- determine the missing measures of sides and angles in problems that involve a uniquely defined oblique triangle if a diagram is given

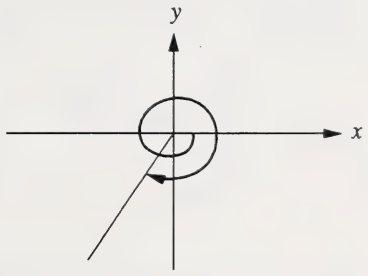
The student demonstrating excellent achievement can:

- determine an expression for any angle that will yield all its possible co-terminal angles
- determine the sine, cosine, and tangent ratios of an angle in standard position given the coordinates of any point, including points with literal coordinates, on its terminal arm
- determine any two of the x-coordinate of a point on the terminal arm of an angle in standard position, the y-coordinate of a point on the terminal arm of an angle in standard position, the measure of the angle, and the distance that the point lies from the origin, given the other two
- determine the measures of all angles between $0°$ and $360°$ that satisfy a given sine, cosine, or tangent ratio
- describe, orally or in writing, the effects of changing both a and b on the graph of the function $y = a \sin b\theta$
- determine the measures of sides and angles in problems that involve right triangles in two dimensions
- given the diagram of three-dimensional objects composed of two or more right triangles, determine the missing measures of sides and angles of the triangles
- determine the missing measures of sides and angles in problems that involve oblique triangles

The student demonstrating acceptable achievement can answer the following types of questions.

SCENARIO: **CONNECTIONS**
Use your understanding that angles and functions can be drawn on a coordinate plane to answer the next questions.

1. An angle in standard position is shown below.



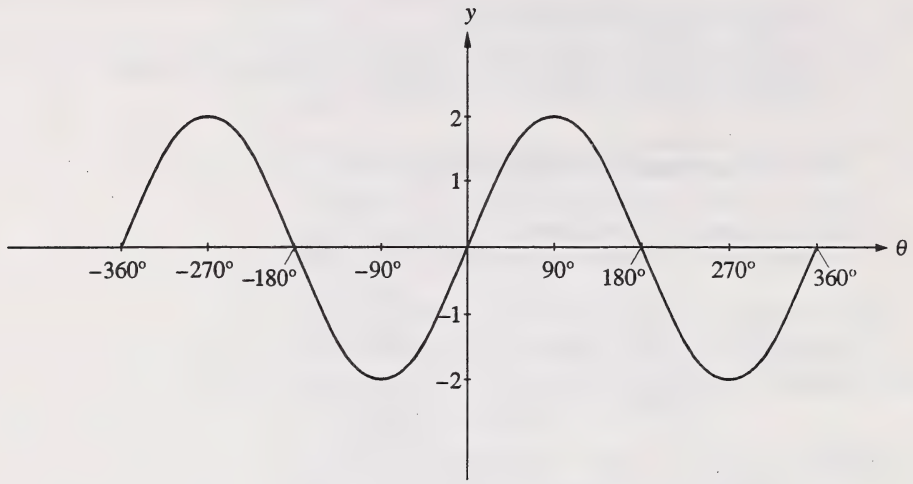
The measure of the principal angle could be

* **A.** 220°
B. –140°
C. 140°
D. –580°

2. If $\tan\theta = -\frac{5}{12}$ and $0° < \theta < 180°$, then $\sin\theta$ equals

* **A.** $\frac{5}{13}$
B. $\frac{13}{12}$
C. $\frac{12}{13}$
D. $\frac{5}{17}$

3. Dale was graphing the sine function $y = a \sin b\theta$ for selected values of a and b on the computer and obtained the graph shown below.



In order to double the amplitude of this graph, Dale should

* **A.** double the value of a

B. double the value of b

C. halve the value of a

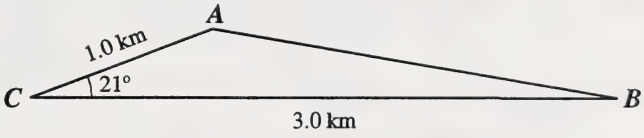
D. halve the value of b

SCENARIO: **ALBERTA INDUSTRIES**

A surveyor for an Alberta industry used aerial photographs and diagrams to solve the following problems.

Written Response – 3 marks

1. The diagram below shows measurements between towns A, B, and C. The distance between Town A and Town C is 1.0 km. The distance between Town C and Town B is 3.0 km and the measure of $\angle ACB$ is 21°.



Find the distance from Town A to Town B to the nearest tenth of a kilometre.

Solution:

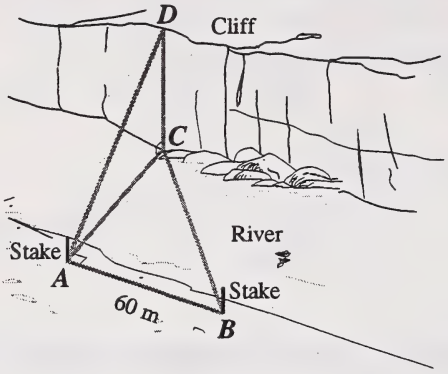
$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\cos C \\ &= 3^2 + 1^2 - 2(3)(1)(\cos 21°) \\ &= 9 + 1 - 5.6015 \\ c &= 2.1 \text{ km} \end{aligned}$$

The distance is 2.1 km.

The student demonstrating excellent achievement can answer the following types of questions.

Written Response – 4 marks

The surveyor drew the following diagram, which shows the measurements used to find the height of a cliff.



Note: $\angle DAC = 20°$
$\angle ABC = 40°$
$\angle ACD = 90°$
$\angle CAB = 90°$
$\overline{AB} = 60\text{m}$

2. Determine the height $\overline{DC}$ of the cliff to the nearest tenth of a metre.

Possible Solution:

The surveyor can use two right triangles ΔABC and ΔACD to solve this problem. The distance across the river, $\overline{AC}$, can be found by:

$$\tan 40° = \frac{\overline{AC}}{60}$$

$$60 \tan 40° = \overline{AC}$$

$$\overline{AC} = 50.3$$

Use ΔABC and the distance across the river, $\overline{AC}$, to find the height of the cliff, $\overline{DC}$.

$$\tan 20° = \frac{\overline{DC}}{50.3}$$

$$\overline{DC} = 50.3 \tan 20°$$

$$\overline{DC} = 18.3$$

The height of the cliff is 18.3 m.

Statistics

Students who achieve the standards can demonstrate the following general outcome upon completion of the unit: *Collect, organize, analyze and draw inferences from bivariate data and from data obtained through "yes/no" surveys.*

Illustration of the general outcome
Given a problem whose solution requires the use of statistics, students should be able to design and administer surveys, collect and organize the results of surveys, draw inferences from surveys including bivariate data and yes/no questions, and determine the confidence intervals for the results of yes/no surveys.

The student demonstrating acceptable achievement can:

- collect and plot bivariate data
- design and administer surveys, collect and organize results, and draw inferences from surveys including bivariate data and yes/no questions
- assess the strengths, weaknesses, and biases of samples
- recognize and describe the apparent correlation between the variables of a bivariate distribution from a scatter plot
- draw 50% and 90% box plots of the results of multiple samples
- plot a line of best fit on a scatter plot using the median fit method
- use charts of 90% box plots to determine the confidence interval of the proportion of yeses in the population
- illustrate the steps or strategies that lead to the solution of problems that require the analysis of statistics studied in Mathematics 33

The student demonstrating excellent achievement can:

- describe, orally or in writing, what is meant by the confidence interval for the proportion of yeses in the population
- develop and use prediction equations for a line of best fit to make inferences for populations
- complete the solution to problems that require the analysis of statistics studied in Mathematics 33

The student demonstrating acceptable achievement can answer the following types of questions.

SCENARIO: **SPORTS AND RECREATION**
Management decisions related to running a sports franchise are often linked to statistics. In the next question, the management would like you to draw an inference about the percentage of patrons who will buy hot dogs.

Numerical Response

1. Out of a sample of 20 randomly selected patrons at a baseball stadium, 8 bought a hot dog during the game. Based on this sample and using a 90% box plot, the percentage of all patrons at the baseball stadium buying a hot dog is between 25% and ____60____ %.

(Record your answer on the answer sheet.)

For broadcast purposes, the management would like you to analyze the number of calls to be routed through Alberta.

1. To relay the results of a ball game, 40 phone calls were placed from Nova Scotia to British Columbia. Thirty-two of these phone calls were routed through telecommunication networks in Alberta. Using a 90% box plot, one could determine that 70% to 85% of all phone calls between Nova Scotia and British Columbia are routed through Alberta.

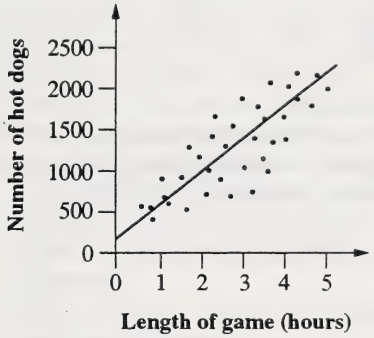
If 120 phone calls are placed between Nova Scotia and British Columbia to relay ball game results, then the number of calls that would likely be routed through Alberta is between

* **A.** 84 and 102
B. 70 and 85
C. 60 and 75
D. 50 and 85

The student demonstrating excellent achievement can answer the following types of questions.

Use the following information to answer the next question.

The scatter plot below shows the relationship between the length of baseball games and the number of hot dogs sold.



Written Response – 4 marks

1. **a.** If (1, 600) and (4, 1800) are two points on the line of **best** fit, then determine the equation of the line of best fit. Use the letter n to represent the number of hot dogs sold and the letter l to represent the length of the game.

Possible Solution:

Note: The slope y-intercept form of a line is $y = mx + b$. To determine the slope (m) and the y-intercept (b) of the line of best fit, start with $n = ml + b$ and build a system of equations.

$$\begin{array}{lrcll} & n & = & ml + b & \\ \text{For 4 hours:} & 1\,800 & = & m(4) + b & \rightarrow \quad 1\,800 = 4m + b \\ \text{For 1 hour:} & 600 & = & m(1) + b & \rightarrow \quad \underline{600 = m + b} \\ & & & & \quad \frac{1\,200}{3} = \frac{3}{3}m \\ & & & & \quad 400 = m \\ & & & & \quad \therefore\ b = 200 \end{array}$$

Since the slope is 400 and the y-intercept is 200, the equation is $n = 400\,l + 200$, where n is the number of hot dogs sold and l is the length of the game in hours.

b. Use the equation that you wrote in part **a** to explain how increasing the length of the game correlates to the number of hot dogs sold.

Possible Solution:
Increasing the length of the game increases the number of hot dogs sold. This is consistent with increasing the value of l in the equation $n = 400l + 200$. As you increase the value of l, the value of n (number of hot dogs sold) increases.

Numerical Response

2. On another day in another stadium, the line of best fit for a scatter plot was determined to be $n = 300\,l + 150$ where n is the number of hot dogs sold and l is the length of the game in hours. For a two-hour game, the number of hot dogs the stadium should expect to sell is _____750_____ .

(Record your answer on the answer sheet.)

Annuities, Mortgages and Loans

Students who achieve the standards can demonstrate the following general outcome upon completion of the unit: *Apply the concepts of ratio, rate, percentage and proportion to annuities, loans, and mortgages.*

Illustration of the general outcome
Extend understanding of rate, ratios, and percentage and proportion to solve real-world problems related to annuities, mortgages, and loans.

The student demonstrating acceptable achievement can:
- recognize that an annuity is a sequence of payments paid in regular intervals for purposes of investment or income
- recognize that a loan is a sum of money that is borrowed to purchase items and is usually repaid in a sequence of payments
- recognize that a mortgage is a special type of loan used to purchase real estate
- given the principal, interest rate, and term, determine the monthly payment for a loan or mortgage
- solve problems that are clearly identified as mortgage, loan, or annuity problems
- given the periodic payment, interest rate, and term, determine the amount of an annuity
- given the periodic payment, number of payments, and interest rate, determine the present value of an annuity
- show that the amount or present value of an annuity is determined by multiplying the amount of a regular payment by a value in a table
- show how the amount or present value of an annuity can be determined using a calculator or spreadsheet

The student demonstrating excellent achievement can:
- given a situation, identify the situation as a present value of an annuity or an amount of an annuity and then solve a related problem
- given the principal, interest rate, and term, find the monthly payment of a loan or mortgage and the interest or the total amount to be repaid
- given a situation, identify it as a situation where a loan, annuity, or mortage would be necessary, and solve the related problem
- given the total amount of an annuity, the interest rate, and term, determine the periodic payment
- given the present value of an annuity, number of payments, and interest rate, determine the periodic payments

The following are examples for using the Annuity and Present Value of an Annuity tables contained in the Data Booklet (Appendix E).

Amount of an Annuity

The amount of the annuity (A) = the amount of a regular payment (R) $\times$ value read from the table ($s_{\overline{n}|i}$), where n is the number of payments and i is the interest rate per period.

Example 1

Determine the amount of an annuity for regular yearly payments of \$50 for 4 years at 6% per annum compounded yearly.

Solution:

Since this is an annuity question, find the amount of the annuity by multiplying the amount of a regular payment by the value ($s_{\overline{n}|i}$) found in the tables.

The amount of the regular payment (R) = 50

Number of payments (n) is 4

Interest per period (i) is 6%

Therefore, from the tables $S_{\overline{n}|i} = 4.63709$

Since the amount of the annuity is $Rs_{\overline{n}|i}$

$= 50(4.63709)$

$= 231.85$

The amount of the annuity is \$231.85.

Example 2

Determine the monthly payments required to produce \$5 000 in 3 years if interest is at 12% per annum.

Solution:

Since this is an annuity question, find the regular payment by dividing the amount of the annuity (A) by the value found in the table ($s_{\overline{n}|i}$).

The amount of the annuity is 5 000

Number of payments (n) $= (3 \times 12) = 36$

Interest per period (i) $= \frac{12\%}{12} = 1\%$

Therefore, from the tables $S_{\overline{n}|i} = 43.50765$

The amount of the annuity is $Rs_{\overline{n}|i}$

Therefore, $5\,000 = R(43.50765)$

$$R = \frac{5\,000}{43.50765}$$

$$= 114.92$$

The monthly payments on this annuity will be \$114.92.

Present Value of an Annuity Formula

The present value of an annuity is the amount of a regular payment (R) $\times$ the value found in the table ($a_{\overline{n}|i}$), where,

n is the number of payments and
i is the interest rate per period.

Example 1

Chris overspent on a charge card account and cut up the card to avoid future debits. Chris must pay \$75 at the end of each month for the next 3 years to settle the debt. If the interest rate is 18% per annum compounded monthly, what is the amount now owing on his account?

Solution:

Since this question asks for the value of the account owing now from a loan, we consider this as a present value question. The present value can be determined by multiplying the amount of a regular payment by the value found in the tables ($a_{\overline{n}|i}$).

Number of payments (n) $= (3 \times 12) = 36$

Interest per period (i) $= \frac{18\%}{12} = 1\frac{1}{2}\%$

Therefore, from the tables $a_{\overline{n}|i} = 27.66068$

The present value is $Ra_{\overline{n}|i}$

Therefore, present value $= 75(27.66068)$

$= 2\,074.55$

The balance owing on the charge card account is \$2 074.55.

Example 2

Determine the regular semi-annual payments that you would receive from an investment of \$20 000, if it is invested in an annuity paying 8% per annum compounded semi-annually for 10 years.

Solution:

This question indicates that you will be receiving regular payments from the \$20 000 that you invest now; therefore, this is a present value question.
The present value is $Ra_{\overline{n}|i}$, where the present value is 20 000, and **R** = ?

Number of payments = (10×2) 20

Interest per period = $\frac{8\%}{2}$ = 4%

Therefore, from the tables $a_{\overline{n}|i}$ = 13.59033

The present value is $Ra_{\overline{n}|i}$

Therefore, 20 000 = R(13.59033)

$$R = \frac{20\,000}{13.59033}$$

= 1 471.63

The investment will pay regular semi-annual payments of \$1 471.63.

The student demonstrating acceptable achievement can answer the following types of questions.

SCENARIO: **CONSUMERISM**
Kelly, a recent Mathematics 33 graduate, is able to use his understanding of annuities, mortages, and loans to determine the answers to many consumer questions. Use your understanding of this area to answer the questions that follow.

1. A mortgage is **best** described as

* **A.** money that is borrowed to purchase real estate
B. funds that are borrowed to purchase items
C. a sequence of payments you receive at regular intervals
D. an amount paid at each interval

Explanation:

Mortgages are used to purchase homes and large investments, therefore, **A.** is the **best** response.

Kelly also decides to save for a new car and would like to determine the total amount that could be saved in 3 years in an annuity.

2. At the beginning of each month, Kelly deposits \$200 in an account that bears interest at 6% per annum compounded monthly. How much money will Kelly have at the end of 3 years?

A. \$7 848.92
* **B.** \$7 906.56
C. \$8 701.53
D. \$12 148.63

Explanation:

Since this is an amount of an annuity question, the amount of an annuity is $Rs_{\overline{n}|i}$, where the amount of the annuity is unknown and $R = 200$

Number of payments $= (3 \times 12) = 36$

Interest per period $= \frac{6\%}{12} = \frac{1}{2}\%$

From the "Amount of an Annuity Table $s_{\overline{n}|i} = 39.53279$

The amount of the annuity is $Rs_{\overline{n}|i}$

Therefore, the amount $= 200(39.53279)$
$= 7\,906.56$

Kelly will have \$7 906.56 in the account at the end of 3 years; therefore, choice **B** is the correct response.

SPECIAL NOTE:
The course of studies recommends that students approach the problems involving annuities using tables and spreadsheets, as opposed to using the sum of a geometric series.

To standardize the use of tables, a data book containing a set of tables approved by teacher validation committees and financial institutions in Alberta is provided in Appendix E.

Kelly's friend Allyson takes out a loan that has a present value of $10 000 to put toward the purchase of a new automobile.

3. If Allyson's loan is for a 3-year term at an interest rate of 12% per annum compounded monthly, she will repay the loan in equal monthly payments of

A. $163.34
B. $208.33
* **C.** $332.14
D. $833.33

Explanation

The present value of the debt is $10 000; therefore, this is a present value question. The present value is $PV = Ra_{\overline{n}|i}$, where the present value is 10 000 and $R = ?$

Number of payments $= (3 \times 12) = 36$

Interest per period $= \frac{12\%}{12} = 1\%$

Therefore, from the tables $a_{\overline{n}|i} = 30.10751$

The present value is $Ra_{\overline{n}|i}$

Therefore, $10\,000 = R(30.10751)$

$$R = \frac{10\,000}{30.10751} = 332.14$$

Allyson's monthly payments will be $332.14; therefore, choice **C** is the correct response.

The student demonstrating excellent achievement can answer the following types of questions.

> *Allyson determined the annual payment required for an annuity to amount to $20 000.*

1. Determine the amount of the annual payment that Allyson needs to make at the beginning of each year for 25 years at 10%/a compounded annually that will amount to $20 000 at the end of the 25th year.

Solution:

Since this is an amount of an annuity question, where the amount of the annuity is $Rs_{\overline{n}|i}$. The amount of the annuity is 20 000.

The number of periods = $(1 \times 25) = 25$

Interest per payment = 10%

Therefore, from the Amount of an Annuity table

$$s_{\overline{n}|i} = 108.18177$$

The amount of the annuity is $Rs_{\overline{n}|i}$

Therefore, $20\,000 = R(108.18177)$

$$R = \frac{20\,000}{108.18177} = 184.87$$

The annual payment is $184.87.

Kelly received a retirement package of \$10 000, which she used to purchase an annuity.

2. Kelly purchased an annuity that paid her a fixed amount at the end of every month for 3 years. If interest rates are 12% per annum, compounded monthly, how much will Kelly receive every month?

Solution:

Kelly is collecting from her investment where the present value of the investment is \$10 000. You need to determine how much she will pay now; therefore, this is a present value question. The present value is $Ra_{\overline{n}|i}$, where the present value is 10 000 and $R =$ _____.

Number of payments = (3×12) = 36

Interest per period $= \frac{12\%}{12}$ = 1%

Therefore, from the tables $a_{\overline{n}|i}$ = 30.10751

The present value is $Ra_{\overline{n}|i}$

Therefore, 10 000 = R(30.10751)

$$R = \frac{10\,000}{30.10751} = 332.14$$

Kelly will receive \$332.14 monthly for three years from her investment of \$10 000.

Appendix D Answer Sheet

Following is a copy of the answer sheet that will be used for the 1996 Mathematics 33 diploma examinations. Teachers are encouraged to copy and use the answer sheet for their examinations in order to familiarize students with it.

LAST NAME

GIVEN NAMES

PERMANENT ADDRESS PHONE

TOWN OR CITY POSTAL CODE

ALBERTA EDUCATION STUDENT ID NUMBER

SEX

DATE OF BIRTH (YEAR/MONTH/DAY)

WRITING CENTRE — CODE

SCHOOL OF ENROLLMENT — CODE

JURISDICTION OF ENROLLMENT — CODE

IMPORTANT DIRECTIONS

1. USE HB PENCIL ONLY.
2. MAKE HEAVY BLACK MARKS THAT FILL IN THE RESPONSE CIRCLES.
3. DO NO MAKE ANY STRAY MARKS ON THIS PAPER.
4. ERASE CLEANLY ANY ANSWER YOU WISH TO CHANGE.

GRADE 12 DIPLOMA EXAMINATION

MATHEMATICS 33

Students First! Student Evaluation

Alberta EDUCATION

PLEASE DO NOT WRITE IN THIS AREA

SERIAL #

MULTIPLE CHOICE

1 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ	17 Ⓐ Ⓑ Ⓒ Ⓓ	25 Ⓐ Ⓑ Ⓒ Ⓓ	33 Ⓐ Ⓑ Ⓒ Ⓓ	41 Ⓐ Ⓑ Ⓒ Ⓓ
2 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ	18 Ⓐ Ⓑ Ⓒ Ⓓ	26 Ⓐ Ⓑ Ⓒ Ⓓ	34 Ⓐ Ⓑ Ⓒ Ⓓ	42 Ⓐ Ⓑ Ⓒ Ⓓ
3 Ⓐ Ⓑ Ⓒ Ⓓ	11 Ⓐ Ⓑ Ⓒ Ⓓ	19 Ⓐ Ⓑ Ⓒ Ⓓ	27 Ⓐ Ⓑ Ⓒ Ⓓ	35 Ⓐ Ⓑ Ⓒ Ⓓ	43 Ⓐ Ⓑ Ⓒ Ⓓ
4 Ⓐ Ⓑ Ⓒ Ⓓ	12 Ⓐ Ⓑ Ⓒ Ⓓ	20 Ⓐ Ⓑ Ⓒ Ⓓ	28 Ⓐ Ⓑ Ⓒ Ⓓ	36 Ⓐ Ⓑ Ⓒ Ⓓ	44 Ⓐ Ⓑ Ⓒ Ⓓ
5 Ⓐ Ⓑ Ⓒ Ⓓ	13 Ⓐ Ⓑ Ⓒ Ⓓ	21 Ⓐ Ⓑ Ⓒ Ⓓ	29 Ⓐ Ⓑ Ⓒ Ⓓ	37 Ⓐ Ⓑ Ⓒ Ⓓ	45 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ	14 Ⓐ Ⓑ Ⓒ Ⓓ	22 Ⓐ Ⓑ Ⓒ Ⓓ	30 Ⓐ Ⓑ Ⓒ Ⓓ	38 Ⓐ Ⓑ Ⓒ Ⓓ	46 Ⓐ Ⓑ Ⓒ Ⓓ
7 Ⓐ Ⓑ Ⓒ Ⓓ	15 Ⓐ Ⓑ Ⓒ Ⓓ	23 Ⓐ Ⓑ Ⓒ Ⓓ	31 Ⓐ Ⓑ Ⓒ Ⓓ	39 Ⓐ Ⓑ Ⓒ Ⓓ	47 Ⓐ Ⓑ Ⓒ Ⓓ
8 Ⓐ Ⓑ Ⓒ Ⓓ	16 Ⓐ Ⓑ Ⓒ Ⓓ	24 Ⓐ Ⓑ Ⓒ Ⓓ	32 Ⓐ Ⓑ Ⓒ Ⓓ	40 Ⓐ Ⓑ Ⓒ Ⓓ	48 Ⓐ Ⓑ Ⓒ Ⓓ

NUMERICAL RESPONSE

1	2	3	4	5	6	7
⊙ ⊙	⊙ ⊙	⊙ ⊙	⊙ ⊙	⊙ ⊙	⊙ ⊙	⊙ ⊙
0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0
1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1
2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2
3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3
4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4
5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5
6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6
7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7
8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8
9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9

8	9	10	11	12	13	14
⊙ ⊙	⊙ ⊙	⊙ ⊙	⊙ ⊙	⊙ ⊙	⊙ ⊙	⊙ ⊙
0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0
1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1
2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2
3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3
4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4
5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5
6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6
7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7
8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8
9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9

Appendix E Data Booklet

The following is the proposed Data Booklet

Mathematics 33 – Draft Formula Sheet

The following information may be useful in writing this examination.

- The general form of a function is
  $y = c \bullet f(x - a) + b$
- The slope of a line joining the two points (x_1, y_1) and (x_2, y_2) is
  $m = \frac{y_2 - y_1}{x_2 - x_1}$
- The equation of a line where m is the slope and b is the y-intercept is
  $y = mx + b$

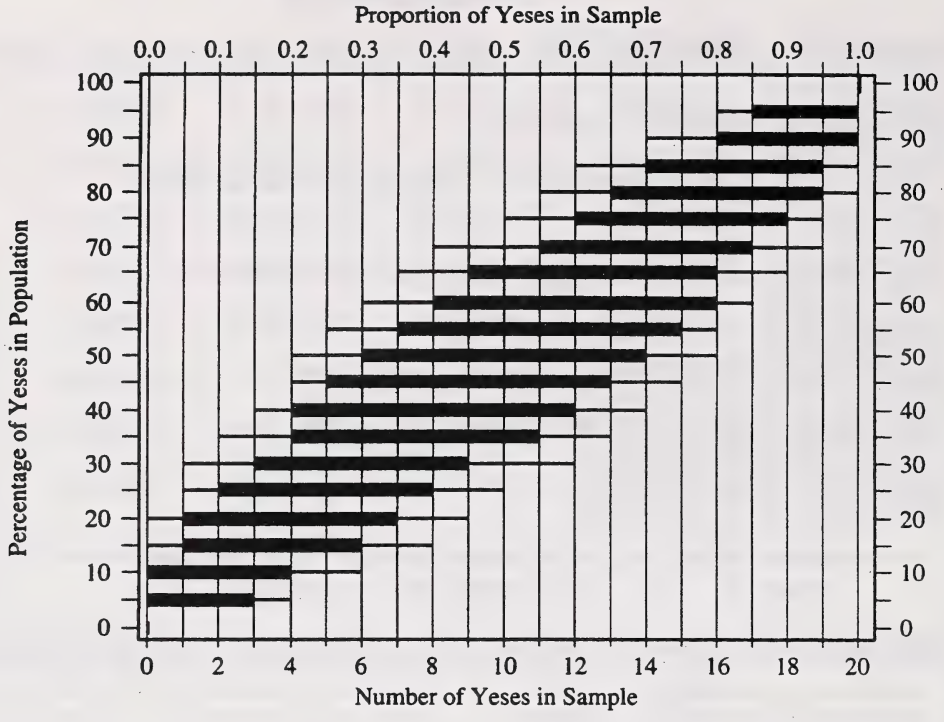
Trigonometry

- $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
- $a^2 = b^2 + c^2 - 2bc \cos A$
- $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

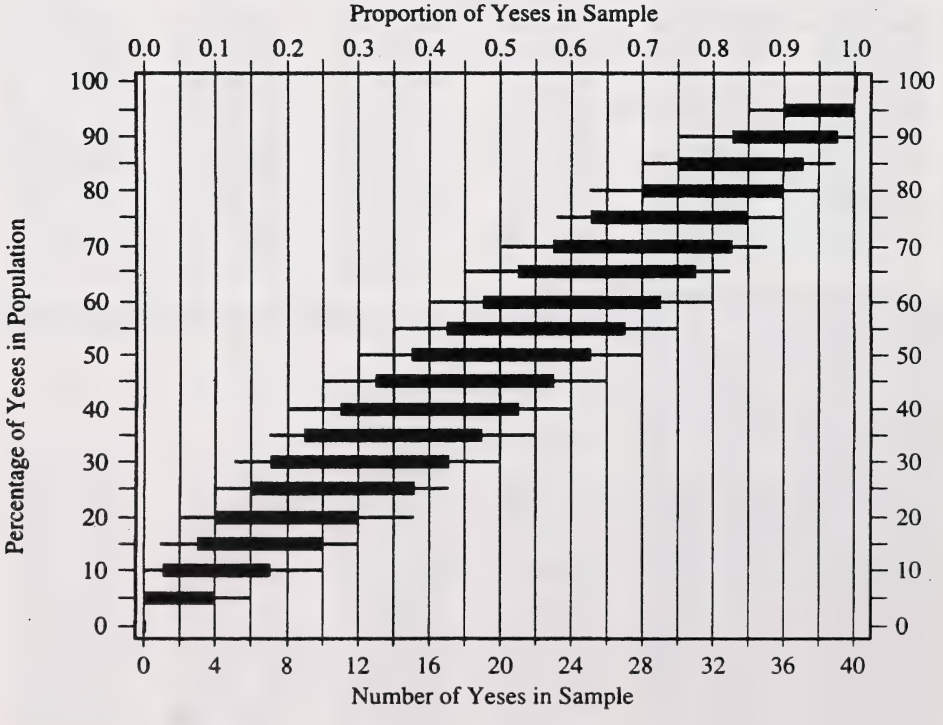
Quadratic Functions and Equations

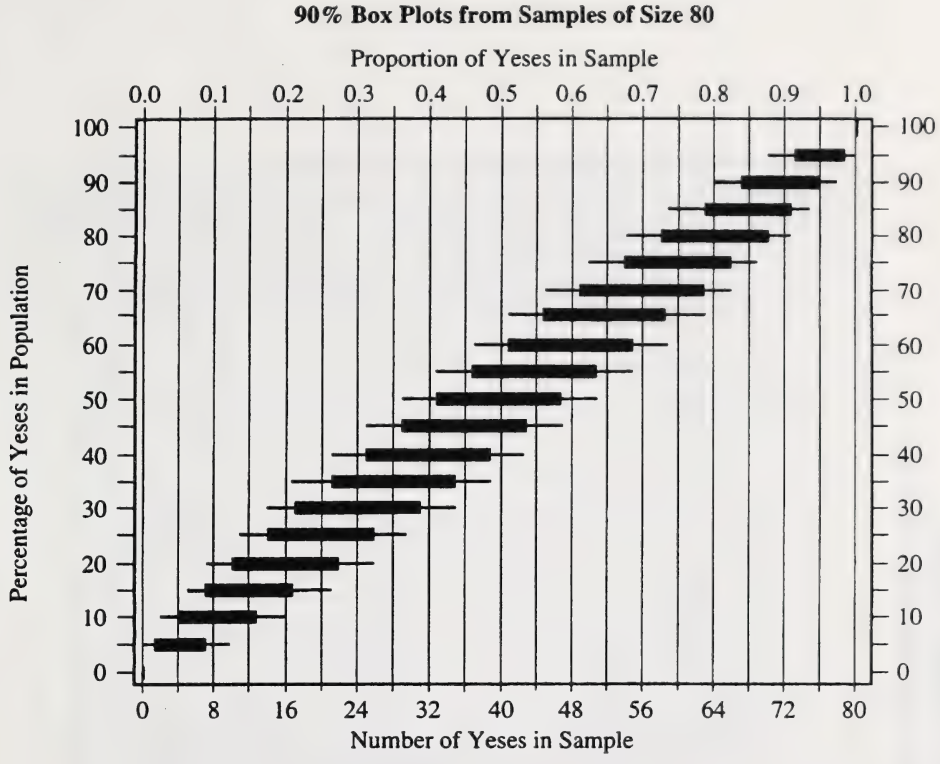
- $y = a(x - h)^2 + k$
- $y = ax^2 + bx + c$
- The roots of the quadratic equation $ax^2 + bx + c = 0$ are
  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a \neq 0$

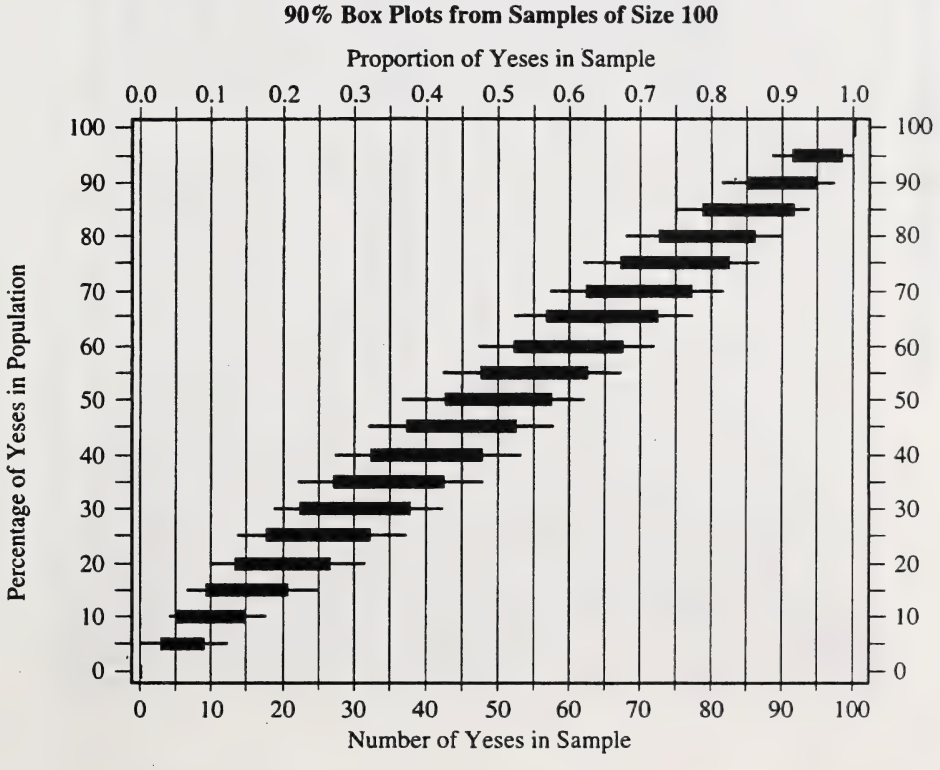
90% Box Plots from Samples of Size 20



90% Box Plots from Samples of Size 40




90% Box Plots from Samples of Size 80
Proportion of Yeses in Sample
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
Percentage of Yeses in Population
100 90 80 70 60 50 40 30 20 10 0
0 8 16 24 32 40 48 56 64 72 80
Number of Yeses in Sample


90% Box Plots from Samples of Size 100
Proportion of Yeses in Sample
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
Percentage of Yeses in Population
100 90 80 70 60 50 40 30 20 10 0
0 10 20 30 40 50 60 70 80 90 100
Number of Yeses in Sample

Monthly Payment On A $1000 Loan

(Includes Principal and Interest Calculated at the End of Each Month)

YEARS	0.5	1	1.5	2	3	4	5	6	7	8	YEARS
RATE IN %											RATE IN %
6.00	169.5950	86.0664	58.2317	44.3206	30.4219	23.4850	19.3328	16.5729	14.6086	13.1414	6.00
6.25	169.7180	86.1814	58.3449	44.4333	30.5353	23.5998	19.4493	16.6912	14.7287	13.2635	6.25
6.50	169.8414	86.2965	58.4582	44.5463	30.6490	23.7150	19.5662	16.8099	14.6495	13.3863	6.50
6.75	169.9633	86.4115	58.5715	44.6593	30.7629	23.8304	19.6835	16.9292	14.9708	13.5096	6.75
7.00	170.0859	86.5268	58.6849	44.7726	30.8771	23.9462	19.8012	17.0490	15.0927	13.6337	7.00
7.25	170.2087	86.6420	58.7986	44.8860	30.9915	24.0624	19.9194	17.1693	15.2152	13.7585	7.25
7.50	170.3313	86.7574	58.9123	44.9995	31.1062	24.1789	20.0379	17.2901	15.3383	13.8839	7.50
7.75	170.4542	86.8729	59.0263	45.1134	31.2212	24.2957	20.1570	17.4114	15.4620	14.0099	7.75
8.00	170.5771	86.9885	59.1404	45.2273	31.3364	24.4129	20.2764	17.5333	15.5862	14.1367	8.00
8.25	170.7000	87.1041	59.2544	45.3414	31.4518	24.5304	20.3963	17.6556	15.7111	14.2641	8.25
8.50	170.8231	87.2198	59.3687	45.4557	31.5675	24.6483	20.5165	17.7784	15.8365	14.3921	8.50
8.75	170.9459	87.3356	59.4831	45.5701	31.6835	24.7665	20.6372	17.9017	15.9625	14.5208	8.75
9.00	171.0688	87.4515	59.5976	45.6847	31.7997	24.8850	20.7584	18.0255	16.0891	14.6502	9.00
9.25	171.1920	87.5675	59.7123	45.7995	31.9162	25.0039	20.8799	18.1499	16.2162	14.7802	9.25
9.50	171.3152	87.6834	59.8271	45.9145	32.0330	25.1232	21.0019	18.2747	16.3440	14.9109	9.50
9.75	171.4382	87.7997	59.9420	46.0296	32.1499	25.2427	21.1242	18.4000	16.4723	15.0422	9.75
10.00	171.5610	87.9159	60.0571	46.1449	32.2672	25.3626	21.2470	18.5258	16.6012	15.1741	10.00
10.25	171.6845	88.0323	60.1723	46.2604	32.3847	25.4828	21.3703	18.6522	16.7307	15.3068	10.25
10.50	171.8078	88.1486	60.2875	46.3760	32.5024	25.6034	21.4939	18.7790	16.8607	15.4400	10.50
10.75	171.9311	88.2651	60.4030	46.4919	32.6205	25.7243	21.6179	18.9063	16.9913	15.5739	10.75

YEARS	0.5	1	1.5	2	3	4	5	6	7	8	YEARS
RATE IN %											RATE IN %
11.00	172.0548	88.3817	60.5185	46.6079	32.7387	25.8455	21.7425	19.0341	17.1225	15.7084	11.00
11.25	172.1779	88.4983	60.6342	46.7240	32.8572	25.9671	21.8673	19.1624	17.2542	15.8436	11.25
11.50	172.3011	88.6150	60.7500	46.8403	32.9760	26.0890	21.9926	19.2911	17.3864	15.9794	11.50
11.75	172.4246	88.7319	60.8660	46.9568	33.0950	26.2113	22.1183	19.4202	17.5193	16.1158	11.75
12.00	172.5486	88.8488	60.9820	47.0735	33.2143	26.3338	22.2444	19.5502	17.6527	16.2528	12.00
12.25	172.6717	88.9658	61.0982	47.1903	33.3338	26.4567	22.3710	19.6804	17.7867	16.3905	12.25
12.50	172.7955	89.0829	61.2146	47.3073	33.4537	26.5800	22.4980	19.8112	17.9213	16.5288	12.50
12.75	172.9192	89.2000	61.3310	47.4245	33.5737	26.7036	22.6253	19.9424	18.0563	16.6677	12.75
13.00	173.0426	89.3172	61.4476	47.5418	33.6939	26.8275	22.7531	20.0741	18.1919	16.8072	13.00
13.25	173.1667	89.4346	61.5643	47.6594	33.8145	26.9518	22.8813	20.2063	18.3282	16.9474	13.25
13.50	173.2902	89.5520	61.6811	47.7770	33.9353	27.0763	23.0099	20.3390	18.4649	17.0882	13.50
13.75	173.4141	89.6694	61.7980	47.8948	34.0563	27.2012	23.1388	20.4721	18.6022	17.2295	13.75
14.00	173.5383	89.7872	61.9152	48.0129	34.1777	27.3265	23.2683	20.6058	18.7400	17.3715	14.00
14.25	173.6619	89.9049	62.0324	48.1311	34.2992	27.4521	23.3981	20.7398	18.8784	17.5141	14.25
14.50	173.7859	90.0225	62.1497	48.2494	34.4210	27.5779	23.5283	20.8744	19.0173	17.6572	14.50
14.75	173.9098	90.1404	62.2672	48.3680	34.5430	27.7042	23.6589	21.0095	19.1568	17.8011	14.75
15.00	174.0339	90.2582	62.3848	48.4866	34.6653	27.8307	23.7899	21.1450	19.2968	17.9454	15.00
15.25	174.1576	90.3763	62.5025	48.6055	34.7879	27.9576	23.9213	21.2810	19.4373	18.0903	15.25
15.50	174.2822	90.4945	62.6204	48.7246	34.9107	28.0849	24.0532	21.4175	19.5784	18.2359	15.50
15.75	174.4063	90.6127	62.7384	48.8438	35.0337	28.2124	24.1854	21.5544	19.7199	18.3821	15.75

Monthly Payment On A $1000 Mortgage

(Includes Principal and Interest Calculated Semi-Annually)

YEARS	1	2	3	4	5	10	15	20	25	YEARS
RATE IN %										RATE IN %
5.00	85.58397	43.84842	29.94787	23.00606	18.84774	10.58149	7.88124	6.57125	5.81605	5.00
5.25	85.69617	43.95809	30.05784	23.11704	18.96000	10.70138	8.00909	6.70691	5.95918	5.25
5.50	85.80834	44.06782	30.16795	23.22824	19.07257	10.82194	8.13798	6.84391	6.10391	5.50
5.75	85.92048	44.17760	30.33965	23.33965	19.18542	10.94318	8.26790	6.98224	6.25022	5.75
6.00	86.03258	44.28743	30.38858	23.45128	19.29857	11.06510	8.39883	7.12188	6.39807	6.00
6.25	86.14465	44.39731	30.49909	23.56311	19.41201	11.18768	8.53076	7.26281	6.54742	6.25
6.50	86.25669	44.50725	30.60973	23.67516	19.52575	11.31093	8.66369	7.40500	6.69824	6.50
6.75	86.36870	44.61724	30.72051	23.78743	19.63977	11.43484	8.79761	7.54844	6.85050	6.75
7.00	86.48067	44.72728	30.83142	23.89990	19.75407	11.55940	8.93249	7.69311	7.00416	7.00
7.25	86.59262	44.83737	30.94246	24.01258	19.86867	11.86461	9.06834	7.83897	7.15919	7.25
7.50	86.70453	44.94751	31.05363	24.12547	19.98355	11.81047	9.20514	7.98602	7.31555	7.50
7.75	86.81641	45.05771	31.16493	24.23857	20.09871	11.93696	9.34287	8.13423	7.47321	7.75
8.00	86.92826	45.16795	31.27635	24.35187	20.21416	12.06409	9.48153	8.28357	7.63213	8.00
8.25	87.04007	45.27824	31.38791	24.46538	20.32988	12.19185	9.62110	8.43404	7.79229	8.25
8.50	87.15186	45.38858	31.49959	24.57909	20.44589	12.32023	9.76158	8.58559	7.95364	8.50
8.75	87.26361	45.49898	31.61140	24.69301	20.56217	12.44924	9.90294	8.37822	8.11614	8.75
9.00	87.37533	45.60942	31.72334	24.80712	20.67873	12.57886	10.04519	8.89189	8.27977	9.00
9.25	87.48701	45.71991	31.83541	24.92144	20.79556	12.70908	10.18830	9.04660	8.44450	9.25
9.50	87.59867	45.83044	31.94760	25.03596	20.91267	12.83991	10.33226	9.20231	8.61028	9.50
9.75	87.71029	45.94103	32.05991	25.15068	21.03005	12.97134	10.47707	9.35899	8.77708	9.75

YEARS	1	2	3	4	5	10	15	20	25	YEARS
RATE IN %										RATE IN %
10.00	87.82188	46.05166	32.17235	25.26560	21.14770	13.10337	10.62270	9.51664	8.94487	10.00
10.25	87.93344	46.16235	32.28491	25.38071	21.26562	13.23598	10.76915	9.67523	9.11362	10.25
10.50	88.04496	46.27307	32.39759	25.49602	21.38380	13.36917	10.91640	9.83473	9.28330	10.50
10.75	88.15646	46.38385	32.51040	25.61152	21.50226	13.50294	11.06445	9.99513	9.45386	10.75
11.00	88.26792	46.49467	32.62332	25.72722	21.62097	13.63729	11.21327	10.15640	9.62529	11.00
11.25	88.37935	46.60554	32.73637	25.84312	21.73995	13.77220	11.36286	10.31851	9.79755	11.25
11.50	88.49074	46.71645	32.84954	25.95920	21.85919	13.90767	11.51320	10.48146	9.97061	11.50
11.75	88.60211	46.82741	32.96283	26.07548	21.97870	14.04369	11.66428	10.64521	10.14443	11.75
12.00	88.71344	46.93842	33.07623	26.19194	22.09846	14.18027	11.81610	10.80974	10.31900	12.00
12.25	88.82474	47.04947	33.18976	26.30860	22.21848	13.31739	11.96862	10.97504	10.49427	12.25
12.50	88.93601	47.16056	33.30340	26.42544	22.33875	14.45505	12.12185	11.14108	10.67023	12.50
12.75	89.04724	47.27170	33.41716	26.54247	22.45928	14.59324	12.27577	11.30784	10.84684	12.75
13.00	89.15844	47.38288	33.53104	26.65969	22.58006	14.73196	12.43137	11.47530	11.02407	13.00
13.25	89.26961	47.49411	33.64503	26.77709	22.70109	14.87120	12.58564	11.64345	11.20191	13.25
13.50	89.38075	47.20638	33.75914	26.89468	22.82238	15.01096	12.74155	11.81225	11.38032	13.50
13.75	89.49185	47.71670	33.87336	27.01245	22.94391	15.15123	12.89811	11.98169	11.55928	13.75
14.00	89.60292	47.82805	33.98770	27.13040	23.06568	15.29200	13.05530	12.15176	11.73876	14.00
14.25	89.71396	47.93945	34.10215	27.24853	23.18771	15.43328	13.21310	12.32243	11.91875	14.25
14.50	89.82497	48.05089	34.21671	27.36685	23.30998	15.57505	13.37150	12.49367	12.09921	14.50
14.75	89.93595	48.16238	34.33139	27.48534	23.43249	15.71730	13.53050	12.66549	12.28012	14.75

Amount Of An Annuity Table

Value of $1 Paid in Each Time Period Calculated at the End of the Period

n	0.5%	1%	1.5%	2%	2.5%	3%	3.5%	4%	4.5%
1	1.00500	1.01000	1.01500	1.02000	1.02500	1.03000	1.03500	1.04000	1.04500
2	2.01503	2.03010	2.04523	2.06040	2.07563	2.09090	2.10623	2.12160	2.13711
3	3.03010	3.06040	3.09090	3.12161	3.15252	3.18363	3.21494	3.24646	3.27822
4	4.05025	4.10101	4.15227	4.20404	4.25633	4.30914	4.36247	4.41632	4.47067
5	5.07550	5.15202	5.22955	5.30812	5.38774	5.46841	5.55015	5.63298	6.71689
6	6.10588	6.21354	6.32299	6.43428	6.54743	6.66246	6.77948	6.89829	7.01911
7	7.14141	7.28567	7.43284	7.58297	7.73612	7.89237	8.05169	8.21423	8.38000
8	8.18212	8.36853	8.55933	8.75463	8.95452	9.15911	9.36850	9.58280	9.80200
9	9.22803	9.46221	9.70272	9.94972	10.20338	10.46388	10.73139	11.00611	11.28822
10	10.27917	10.56683	10.86326	11.16872	11.48347	11.80780	12.14199	12.48635	12.84111
11	11.33556	11.68250	12.04121	12.41209	12.79555	13.19203	13.60196	14.02581	14.46400
12	12.39724	12.80933	13.23683	13.68033	14.14044	14.61779	15.11303	15.62684	16.15978
13	13.46423	13.94742	14.45038	14.97394	15.51895	16.08632	16.67698	17.29191	17.93200
14	14.53655	15.09690	15.68214	16.29342	16.93193	17.59891	18.29568	19.02359	19.78400
15	15.61423	16.25786	16.93237	17.63929	18.38022	19.15688	19.97103	20.82453	21.71911
16	16.69730	17.43044	18.20136	19.01207	19.86473	20.76159	21.70502	22.69751	23.74156
17	17.78579	18.61475	19.48938	20.41231	21.38635	22.41444	23.49969	24.64541	25.85489
18	18.87972	19.81090	20.79672	21.84056	22.94601	24.11687	25.35718	26.67123	28.06333
19	19.97912	21.01900	22.12367	23.29737	24.54466	25.87037	27.27968	28.77808	30.37111
20	21.08401	22.23919	23.47052	24.78332	26.18327	27.67649	29.26947	30.96920	32.78289
21	22.19443	23.47159	24.83758	26.29898	27.86286	29.53678	31.32890	33.24797	35.30311
22	23.31040	24.71630	26.22514	27.84496	29.58443	31.45288	33.46041	35.61789	37.93667
23	24.43196	25.97346	27.63352	29.42186	31.34904	33.42647	35.66653	38.08260	40.68889
24	25.55912	27.24320	29.06302	31.03030	33.15776	35.45926	37.94986	40.64591	43.56489
25	26.69191	28.52563	30.51397	32.67091	35.01171	37.55304	40.31310	43.31174	46.57022
26	27.83037	29.82089	31.98668	34.34432	36.91200	39.70963	42.75906	46.08421	49.71089
27	28.97452	31.12910	33.48148	36.05121	38.85980	41.93092	45.29063	48.96728	52.99289
28	30.12439	32.45039	34.99870	37.79223	40.85630	44.21885	47.91080	51.96629	56.42244
29	31.28002	33.78489	36.53868	39.56808	42.90270	46.57546	50.62268	55.08494	60.00644
30	32.44142	35.13274	38.10176	41.37944	45.00027	49.00268	53.42947	58.32934	63.75178
31	33.60862	36.49408	39.68829	43.22703	47.15028	51.50276	56.33450	61.70147	67.66556
32	34.78167	37.86901	41.29861	45.11157	49.35403	54.07784	59.34121	65.20953	71.75556
33	35.96058	39.25770	42.93309	47.03380	51.61289	56.73018	62.45315	68.85791	76.02956
34	37.14538	40.66028	44.59209	48.99448	53.92821	59.46208	65.67401	72.65222	80.49600
35	38.33611	42.07688	46.27597	50.99437	56.30141	62.27594	69.00760	76.59831	85.16311
36	39.53279	43.50765	47.98511	53.03425	58.73395	65.17422	72.45787	80.70225	90.04044

Amount Of An Annuity Table

Value of $1 Paid in Each Time Period Calculated at the End of the Period

n	5%	6%	7%	8%	9%	10%	12%	14%
1	1.05000	1.06000	1.07000	1.08000	1.09000	1.10000	1.12000	1.14000
2	2.15250	2.18360	2.21490	2.24640	2.27810	2.31000	2.37440	2.43960
3	3.31013	3.37462	3.43994	3.50611	3.57313	3.64100	3.77933	3.92114
4	4.52563	4.63709	4.75074	4.86660	4.98471	5.10510	5.35285	5.61010
5	5.80191	5.97532	6.15329	6.33593	6.52333	6.71561	7.11519	7.53552
6	7.14201	7.39384	7.65402	7.92280	8.20043	8.48717	9.08901	9.73049
7	8.54911	8.89747	9.25980	9.63663	10.02847	10.43589	11.29969	12.23276
8	10.02656	10.49132	10.97799	11.48756	12.02104	12.57948	13.77566	15.08535
9	11.57789	12.18079	12.81645	13.48656	14.19293	14.93742	16.54874	18.33730
10	13.20679	13.97164	14.78360	15.64549	16.56029	17.53117	19.65458	22.04452
11	14.91713	15.86994	16.88845	17.97713	19.14072	20.38428	23.13313	26.27075
12	16.71298	17.88214	19.14064	20.49530	21.95338	23.52271	27.02911	31.08865
13	18.59863	20.01507	21.55049	23.21492	25.01919	26.97498	31.39260	36.58107
14	20.57856	22.27597	24.12902	26.15211	28.36092	30.77248	36.27971	42.84241
15	22.65749	24.67253	26.88805	29.32428	32.00330	34.94973	41.75328	49.98035
16	24.84037	27.21288	29.84022	32.75023	35.97370	39.54470	47.88367	58.11760
17	27.13238	29.90565	32.99903	36.45024	40.30134	44.59917	54.74971	67.39407
18	29.53900	32.75999	36.37896	40.44626	45.01846	50.15909	62.43968	77.96923
19	32.06595	35.78559	39.99549	45.76196	50.16012	56.27500	71.05244	90.02493
20	34.71925	38.99273	43.86518	49.42292	55.76453	63.00250	80.69874	103.76842
21	37.50521	42.39229	48.00574	54.45676	61.87334	70.40275	91.50258	119.43600
22	40.43048	45.99583	52.43614	59.89330	68.53194	78.54302	103.60289	137.29704
23	43.50200	49.81558	57.17667	65.76476	75.78981	87.49733	117.15524	157.65862
24	46.72710	53.86451	62.24904	72.10594	83.70090	97.34706	132.33387	180.87083
25	50.11345	58.15638	67.67647	78.95442	92.32398	108.18177	149.33393	207.33274
26	53.66913	62.70577	73.48382	86.35077	101.72313	120.09994	168.37401	237.49933
27	57.40258	67.52811	79.69769	94.33883	111.96822	133.20994	189.69889	271.88923
28	61.32271	72.63980	86.34653	102.96594	123.13536	147.63093	213.58275	311.09373
29	65.43885	78.05819	93.46079	112.28321	135.30754	163.49402	240.33268	355.78685
30	69.76079	83.80168	101.07304	122.34587	148.57522	180.94342	270.29261	406.73701
31	74.29883	89.88978	109.21815	133.21354	163.03699	200.13777	303.84772	464.82019
32	79.06377	96.34316	117.93343	144.95062	178.80032	221.25154	341.42945	531.03501
33	84.06697	103.18375	127.25776	157.62667	195.98234	244.47670	383.52098	606.51991
34	89.32031	110.43478	137.23688	171.31680	214.71075	270.02437	430.66350	692.57270
35	94.83632	118.12087	147.91346	186.10215	235.12472	298.12881	483.46312	790.67288
36	101.62814	126.26812	159.33740	202.07032	257.37595	329.03949	542.59869	902.50708

Present Value Of An Annuity Table

Present Value of $1 Received in Each Time Period

n	0.5%	1%	1.5%	2%	2.5%	3%	3.5%	4%	4.5%
1	0.99502	0.99010	0.98522	0.98039	0.97561	0.97087	0.96618	0.96154	0.95689
2	1.98510	1.97046	1.95588	1.94156	1.92742	1.91347	1.89969	1.88609	1.87267
3	2.97025	2.94099	2.91220	2.88388	2.85602	2.82861	2.80164	2.77509	2.74911
4	3.95050	3.90197	3.85438	3.80773	3.76197	3.71710	3.67308	3.62990	3.58756
5	4.92587	4.85343	4.78265	4.71346	4.64583	4.57971	4.51505	4.45182	4.39000
6	5.89638	5.79548	5.69719	5.60143	5.50813	5.41719	5.32855	5.24214	5.15778
7	6.86207	6.72819	6.59821	6.47199	6.34939	6.23028	6.11454	6.00205	5.89267
8	7.82296	7.65168	7.48593	7.32548	7.17014	7.01969	6.87396	6.73274	6.59578
9	8.77906	8.56602	8.36052	8.16224	7.97087	7.78611	7.60769	7.43533	7.26867
10	9.73041	9.47130	9.22218	8.98259	8.75206	8.53020	8.31661	8.11090	7.91267
11	10.67703	10.36763	10.07112	9.78685	9.51420	9.25266	9.00155	8.76048	8.52889
12	11.61893	11.25508	10.90751	10.57534	10.25776	9.95400	9.66333	9.38507	9.11867
13	12.55615	12.13374	11.73153	11.34837	10.98318	10.63496	10.30274	9.98565	9.68289
14	13.48871	13.00370	12.54338	12.10625	11.69091	11.29607	10.92052	10.56312	10.22289
15	14.41662	13.86505	13.34323	12.84926	12.38138	11.93794	11.51741	11.11839	10.73956
16	15.33993	14.71787	14.13126	13.57771	13.05500	12.56110	12.09412	11.65230	11.23400
17	16.25863	15.56225	14.90765	14.29187	13.71220	13.16612	12.65132	12.16567	11.70711
18	17.17277	16.39827	15.67256	14.99203	14.35336	13.75351	13.18968	12.65930	12.16000
19	18.08236	17.22601	16.42617	15.67846	14.97889	14.32380	13.70984	13.13394	12.59333
20	18.98742	18.04555	17.16864	16.35143	15.58916	14.87747	14.21240	13.59033	13.00778
21	19.88798	18.85698	17.90014	17.01121	16.18455	15.41502	14.69797	14.02916	13.40467
22	20.78406	19.66038	18.62082	17.65805	16.76541	15.93692	15.16712	14.45112	13.78444
23	21.67568	20.45582	19.33086	18.29220	17.33211	16.44361	15.62041	14.85684	14.14778
24	22.56287	21.24339	20.03041	18.91393	17.88499	16.93554	16.05837	15.24696	14.49533
25	23.44564	22.02316	20.71961	19.52346	18.42438	17.41315	16.48151	15.62208	14.82822
26	24.32402	22.79520	21.39863	20.12104	18.95061	17.87684	16.89035	15.98277	15.14667
27	25.19803	23.55961	22.06762	20.70690	19.46401	18.32703	17.28536	16.32959	15.45133
28	26.06769	24.31644	22.72672	21.28127	19.96489	18.76411	17.66702	16.66306	15.74289
29	26.93302	25.06579	23.37608	21.84438	20.45350	19.18845	18.03577	16.98371	16.02178
30	27.79405	25.80771	24.01584	22.39646	20.93023	19.60044	18.39205	17.29203	16.28889
31	28.65080	26.54229	24.64615	22.93770	21.39540	20.00043	18.73628	17.58849	16.55089
32	29.50328	27.26959	25.26714	23.46834	21.84918	20.38877	19.06887	17.87355	16.78889
33	30.35153	27.98969	25.87895	23.98856	22.29181	20.76579	19.39021	18.14765	17.02289
34	31.19555	28.70267	26.48173	24.49859	22.72379	21.13184	19.70068	18.41120	17.24867
35	32.03537	29.40858	27.07559	24.99862	23.14516	21.48722	20.00066	18.66461	17.46089
36	32.87102	30.10751	27.66068	25.48884	23.55625	21.83225	20.29049	18.90828	17.66600

Present Value Of An Annuity Table

Present Value of $1 Received in Each Time Period

n	5%	6%	7%	8%	9%	10%	12%	14%	16%
1	0.95238	0.94340	0.93458	0.92593	0.91743	0.90909	0.89283	0.87721	0.86206
2	1.85941	1.83339	1.80802	1.78326	1.75911	1.73554	1.69008	1.64664	1.60525
3	2.72325	2.67301	2.62432	2.57710	2.53129	2.48685	2.40183	2.32164	2.24588
4	3.54595	3.46511	3.38721	3.31213	3.23972	3.16987	3.03733	2.91371	2.79819
5	4.32948	4.21236	4.10020	3.99271	3.88965	3.79079	3.60475	3.43307	3.27431
6	5.07569	4.91732	4.76654	4.62288	4.48592	4.35526	4.11142	3.88864	3.68475
7	5.78637	5.58238	5.38929	5.20637	5.03295	4.86842	4.56375	4.28829	4.03856
8	6.46321	6.20979	5.97130	5.74664	5.53482	5.33493	4.96767	4.63886	4.34356
9	7.10782	6.80169	6.51523	6.24689	5.99525	5.75902	5.32825	4.94636	4.60656
10	7.72173	7.36009	7.02358	6.71008	6.41766	6.14457	5.65025	5.21614	4.83325
11	8.30641	7.88687	7.49867	7.13896	6.80519	6.49506	5.93767	5.45271	5.02863
12	8.86325	8.38384	7.94269	7.53608	7.16073	6.81369	6.19433	5.66029	5.19713
13	9.39357	8.85268	8.35765	7.90378	7.48690	7.10336	6.42358	5.84236	5.34231
14	9.89864	9.29498	8.74547	8.24424	7.78615	7.36669	6.62817	6.00207	5.46750
15	10.37966	9.71225	9.10771	8.55948	8.06069	7.60608	6.81083	6.14214	5.57544
16	10.83777	10.10590	9.44665	8.85137	8.31256	7.82371	6.97398	6.26507	5.66850
17	11.27407	10.47726	9.76322	9.12164	8.54363	8.02155	7.11958	6.37286	5.74869
18	11.68959	10.82760	10.05909	9.37189	8.75563	8.20141	7.24967	6.46743	5.81788
19	12.08532	11.15812	10.33560	9.60360	8.95011	8.36492	7.36575	6.55036	5.87744
20	12.46221	11.46902	10.59401	9.81815	9.12855	8.51356	7.46942	6.62314	5.92881
21	12.82115	11.76408	10.83553	10.01680	9.29224	8.64869	7.56200	6.68693	5.97313
22	13.16300	12.04158	11.06124	10.20074	9.44243	8.77154	7.64467	6.74293	6.01131
23	13.48857	12.30338	11.27219	10.37106	9.58021	8.88322	7.71842	6.79207	6.04425
24	13.79864	12.55036	11.46933	10.52876	9.70661	8.98474	7.78433	6.83514	6.07263
25	14.09394	12.78336	11.65358	10.67478	9.82252	9.07704	7.84317	6.87293	6.09706
26	14.37519	13.00317	11.82578	10.80998	9.92897	9.16095	7.89567	6.90607	6.11819
27	14.64303	13.21053	11.98671	10.93516	10.02658	9.23722	7.94258	6.93514	6.13638
28	14.89813	13.40616	12.13711	11.05108	10.11613	9.30657	7.98442	6.96064	6.15206
29	15.14107	13.59072	12.27767	11.15841	10.19228	9.36961	8.02183	6.98307	6.16556
30	15.37245	13.76483	12.40904	11.25778	10.27365	9.42691	8.05517	7.00264	6.17719
31	15.59281	13.92909	12.53181	11.34980	10.34280	9.47901	8.08500	7.01986	6.18725
32	15.80268	14.08404	12.64656	11.43500	10.40624	9.52638	8.11158	7.03500	6.19588
33	16.00255	14.23023	12.75379	11.51389	10.46444	9.56943	8.13533	7.04821	6.20388
34	16.19290	14.36814	12.85401	11.58693	10.51784	9.60857	8.15658	7.05986	6.20981
35	16.37419	14.49825	13.94767	11.65457	10.56682	9.64416	8.17550	7.07007	6.21531
36	16.54685	14.62099	13.05321	11.71719	10.61176	9.67651	8.19242	7.07900	6.22013

Appendix F

Explanation of Mathematical Understandings

Procedures

The assessment of students' knowledge of *mathematical procedures* should provide evidence that they can:

- recognize when a procedure is appropriate
- give reasons for the steps in a procedure
- reliably and efficiently execute procedures
- verify the results of procedures empirically (e.g., using models) or analytically
- recognize correct and incorrect procedures
- generate new procedures and extend or modify familiar ones
- appreciate the nature and role of procedures in mathematics

It is important that students know how to execute mathematical procedures reliably and efficiently; a knowledge of procedures involves much more than simple execution. Students must know when to apply them, why they work, and how to verify that they have given a correct answer; they also must understand concepts underlying a procedure and the logic that justifies it. Procedural knowledge also involves the ability to differentiate those procedures that work from those that do not and the ability to modify them or create new ones. Students must be encouraged to appreciate the nature and role of procedures in mathematics; that is, they should appreciate that procedures are created or generated as tools to meet specific needs in an efficient manner and thus can be extended or modified to fit new situations. The assessment of students' procedural knowledge, therefore, should not be limited to an evaluation of their facility in performing procedures: it should emphasize all the aspects of procedural knowledge addressed in this standard.

Concepts

The assessment of students' knowledge and understanding of *mathematical concepts* should provide evidence that they can:

- label, verbalize, and define concepts
- identify and generate examples and non-examples
- use models, diagrams, and symbols to represent concepts
- translate from one mode of representation to another
- recognize the various meanings and interpretations of concepts
- identify properties of a given concept and recognize conditions that determine a particular concept
- compare and contrast concepts

In addition, assessment should provide evidence of the extent to which students have integrated their knowledge of various concepts.

An understanding of mathematical concepts involves more than mere recall of definitions and recognition of common examples; it encompasses the broad range of abilities identified in this standard. Assessment, too, must address these aspects of conceptual understanding. Assessment tasks should focus on students' abilities to discriminate between the relevant and the irrelevant attributes of a concept in selecting examples and non-examples, to represent concepts in various ways, and to recognize students' various meanings. Tasks that ask students to apply information about a given concept in novel situations provide strong evidence of students' knowledge and understanding of that concept. Problems designed to elicit information about students' misconceptions can provide information useful in planning or modifying instruction.

Problem Solving

The assessment of students' ability to use mathematics in *solving problems* should provide evidence that they can:

- formulate problems
- apply a variety of strategies to solve problems
- solve problems
- verify and interpret results
- generalize solutions

Students' ability to solve problems develops over time as a result of extended instruction, opportunities to solve many kinds of problems, and encounters with real-world situations. Students' progress should be assessed systematically, deliberately, and continually to effectively influence their confidence and ability to solve problems in various contexts. Assessments should determine students' ability to perform all aspects of problem solving. Evidence about their ability to ask questions, use given information, and make conjectures is essential to determine if they can formulate problems. Assessments also should yield evidence of students' use of strategies and problem-solving techniques and of their ability to verify and interpret results. Finally, because the power of mathematics is derived, in part, from its generalizability, this aspect of problem solving should be assessed as well.

From *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, 1989, p. 209, p. 223, p. 228.

Appendix G

Generalized Scoring Guide for Extended-Response Items

The following is a generalized scoring guide from which examination item-writers develop holistic scoring guides. Teachers are encouraged to copy or cutomize this guide for their own use.

Scale Score	Descriptor
5	Complete answer, with supporting detail given. Steps are presented clearly and in a logical order. Final answers are correct, and the communication is readily understandable.
4	Complete answer, but with minor errors present. Final answers may be incorrect, and/or the communication may lack some clarity.
3	**Either** a partial answer that is correct and that represents a major step in the solution of the item **or** a complete answer that has one major **or** several minor errors present but that indicates the full intended scope of the item.
2	**Either** a partial answer that is correct except for minor errors and that represents a major step in the solution of the item **or** a complete answer that has one major **and** many minor errors present but that indicates the intended scope of the item.
1	**Either** a correct answer with no supporting detail **or** a significant start has been made to the problem which, if continued, would lead to a successful conclusion to a major step in the solution of the item.
0	**Either** off topic **or** an incorrect answer with no supporting detail **or** a fatal error present. Fatal errors, whether conceptual or procedural, are those errors that destroy the integrity of the original item, often making the item completely trivial **or** a blank paper.
NR	a blank paper, no response

This scoring guide can be applied to an item that is at the acceptable standard and/or at the standard of excellence.

Note: Minor error means —calculation error
—communication error
—missing or incorrect units were applicable

Appendix H

Instructions for Completing the Numerical-Response Questions

- **Use an HB pencil only** for machine-scored answer sheet.
- If you wish to change an answer, erase **all** traces of your first answers.
- Read the question carefully. Consider all numbers used in the questions to be exact real numbers and not the result of a measurement.
- Record your answer on the answer sheet provided by writing it in the boxes and then filling in the corresponding circles.
- **Enter the first digit of your answer in the left-hand box and leave any unused boxes blank.**

Sample Questions and Solutions for Numerical Response

The value of tan 35° to the nearest tenth is _____.
Value: 0.7002
Value to be recorded: 0.7

Record 0.7 on the answer sheet →

The y-intercept for the quadratic function $y = 2x^2 + 7x + 32$ is _____.
Value: 32
Value to be recorded: 32

Record 32 on the answer sheet →

Appendix I

Directing Words

Discuss
The word "discuss" will not be used as a directing word on math and science diploma examinations because it is not used consistently to mean a single activity.

The following words are specific in meaning.

Contrast/Distinguish
Point out the *differences* between two things that have similar or comparable natures.

Compare
Show the character or relative values of two things by pointing out their *similarities* and *differences*.

Conclude
State a logical end based on reasoning and/or evidence.

Criticize
Point out the *merits* and *demerits* of an item or issue.

Define
Provide the essential qualities or meaning of a word or concept. Make distinct and clear by marking out the limits.

Describe
Give a written account or represent the characteristics of something by a figure, model, or picture.

Design/Plan
Construct a plan, i.e, detailed sequence of actions, for a specific purpose.

Enumerate
Specify one by one or list in concise form and according to some order.

Evaluate
Give the significance or worth of something by identifying the good and bad points or the advantages and disadvantages.

Explain
Make clear what is not immediately obvious or entirely known; give the cause of or reason for; make known in detail.

How
Show in what manner or way, with what meaning.

Hypothesize
Form a tentative proposition intended as a possible explanation for an observed phenomenon, i.e., a possible cause for a specific effect. The proposition should be testable logically and/or empirically.

Identify
Recognize and select as having the characteristics of something.

Illustrate
Make clear by giving an example. The form of the example must be specified in the question; i.e., word description, sketch, or diagram.

Infer
Form a generalization from sample data; arrive at a conclusion by reasoning from evidence.

Interpret
Tell the meaning of something; present information in a new form that adds meaning to the original data.

Justify/Show How
Show reasons for or give facts that support a position.

Outline
Give, in an organized fashion, the essential parts of something. The form of the outline must be specified in the question; i.e., lists, flow charts, concept maps.

Predict
Tell in advance on the basis of empirical evidence and/or logic.

Prove
Establish the truth, validity, or genuineness of something by giving factual evidence or logical reasons.

Relate
Show logical or causal connection between things.

Solve
Give a solution for a problem, i.e., explanation in words and/or numbers.

Summarize
Give a brief account of the main points.

Trace
Give a step-by-step description of the development.

Why
Show the cause, reason, or purpose.

Appendix J

Preparing to Write a Mathematics 33 Diploma Examination—What every student should do before the examinations!

1. Prepare a course review schedule:
   - design your schedule for the two-week period (minimum) before the examination
   - divide the course material into sections and indicate on the schedule the time blocks to be devoted to each section
   - take into account the examination blueprint available from your teacher (Diploma Examinations Program Information Bulletin for the course.)
     Note that course units are not equally weighted on the diploma examination
   - take into account units/concepts that you find most difficult; i.e., allocate more time for the review of these

2. Obtain and review examination schedules, rules, and policies:
   - record the time and place of writing
   - note minimum and maximum writing times permitted
   - prepare to remain in the examination room for at least 2.5 h (Kleenex, cough drops, etc.)
   - identify materials allowed for writing each examination, such as pencils, pens, calculators, mathematical instruments, and clear plastic ruler

3. Identify and collect examples of each type of question that will be asked:
   - obtain a copy of the relevant information contained in the *Mathematics 33 Information Bulletin, Diploma Examinations Program* (available from your teacher)
   - learn the meanings of key "directing" words such as compare, describe, evaluate, explain, illustrate, interpret, justify, prove, and solve

4. Make summaries and point form outlines:
   - distinguish between major concepts and factual details
   - identify essential skills that can be assessed on paper and pencil tests
   - review project results and procedures— identify connections between project reports, class notes, and textbook
   - anticipate examples of connections between concepts and the "real world"
   - prepare a glossary of important subject terminology
   - review the data booklet for Mathematics 33, and review formulas and equations if applicable
   - link each formula or equation with a calculation done on a previous test or assignment
   - identify any restriction on the use of each formula or equation

5. Use memory aids such as:
   - colour coding, underlining, highlighting, jotting key words in margins
   - numbering points to be memorized
   - grouping word and idea associations
   - reading aloud key words, expressing key words in your own words

6. Review the different question formats and the instructions on how to answer these questions.

Appendix K

Suggestions for Students When Writing a Mathematics 33 Diploma Examination—What every student should know when writing examinations!

1. Do not be afraid to answer each question even if you are not sure of the correct solution to the problem. A penalty is NOT given for guessing on the machine-scored section the exam. Partial marks are often awarded for incomplete answers in the written-response section of the exam.

2. If you are stuck on a question, mark the alternatives that you know are incorrect and choose from the ones that are left using logical guessing strategy. Think of the questions as challenges and cultivate a positive attitude about your ability to answer them.

3. Scan the sets of questions of the examination before answering a particular question. The questions in one set of the examination may jog your memory about a question in another set.

4. When first reading a multiple-choice question, locate and circle key words to help clarify the meaning of the question. Then hide the alternatives and try to formulate an answer of your own. Your answer may be very close to the correct alternative.

5. If a multiple-choice question involves a calculation, do the calculation and select the alternative that is closest to your answer. A multiple-choice calculation is usually short. If you cannot do it in five minutes, your method is either inappropriate or incorrect. Go on.

6. Diagrams on examinations are often labelled with numbers or letters. It may be useful to write in the names of the labelled structures or features that you can identify.

7. When reading graphs, use a clear plastic ruler to more accurately extrapolate or interpolate data.

8. Have a good reason for changing an answer. Do not change an answer on a hunch. Do not waste your time looking for patterns of As, Bs, Cs, or Ds in multiple-choice answers. There are none.

9. You may not have time to write and edit a complete rough copy for each written-response question, but you should prepare an outline of your answer and use it as a guide when writing your good copy.

10. When completing a written-response question, keep in mind the reader of your response. The reader will want to know how well you:
    - understand the problem or the mathematical concept
    - can correctly use the mathematics involved
    - can use problem-solving strategies and explain your answer and procedures
    - can communicate your solutions and mathematical ideas

11. Rewriting a statement of the question is often a good way to begin a written response. Conclude with a summary statement. Be sure you have clearly explained all assumptions and have verified your conclusions.

12. Keep track of the time and pace yourself. Put a check mark by items that you are uncertain about and return to them if there is time at the end of the examination.